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A SKEPTICAL NOTE ON THE ROLE OF CONSTANT ELASTICITY OF SUBSTITUTION IN LABOR INCOME SHARE DYNAMICS

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Abstract

The constancy of the elasticity of factor substitution (σ) makes its role as a driver of the labor income share exogenous. The constant elasticity of substitution (CES) (Arrow et al., 1961) production function has predominantly been used to support this causal relationship. This paper argues that (i) capital-labor ratio determines the value of σ, and (ii) both capital-labor ratio and σ vary over time. I use a variable elasticity of substitution (VES) production framework that allows both labor income share and σ to change over time. Statistically significant empirical support is provided using the Japanese industrial productivity (JIP) data. This suggests that the CES model may not be an ideal choice to examine the factor income share dynamics.

Keywords: substitution elasticity, labor income share, production function parameters

JEL Classification: E21, E22, E25

Contents

1. INTRODUCTION

"…the production function has been a powerful tool of miseducation."

The Production Function and the Theory of Capital

Joan Robinson (1953–54, page 81)

The elasticity of factor substitution plays an important role in the analysis of factor income shares. For example, the assumption of a non-unitary elasticity of substitution (σ_{KL}) between capital and labor explains the changes in the labor income share over time. A production technology that shows how the allocative efficiency of factor inputs relates to the rate of productivity per worker, on the other hand, governs the characteristics of the elasticity of factor substitution. More than a half-century ago, Liu and Hildebrand (1965) suggested a three-variable relationship between value added per unit of labor ($\frac{Y}{L}$), the wage rate (W) and the capital–labor ratio ($\frac{K}{L}$), which has, since then, served as the basis of estimating a production framework. Equation (1) replicates this relationship in a log-linear form, including a constant term (β_0) and an error term (ε):

$$log\frac{Y}{L} = \beta_0 + \beta_1 logW + \beta_2 log\frac{K}{L} + \varepsilon \quad (1)$$

Arrow et al. (1961) assumed $\beta_2 = 0$, and propounded the constant elasticity of substitution (CES) production function based on the goodness of fit of the empirical relationship between $log\frac{Y}{L}$ and $logW$. The CES production function has, since then, become a prominent model in the studies of economic theory (equation 2). It is a generalized version of the Cobb–Douglas (CD) production function (Cobb and Douglas 1928),[1] and reduces to the CD form when $\sigma = 1$.

$$Y = A\left[\theta K^{\frac{\sigma-1}{\sigma}} + (1-\theta)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}};\ \beta_2 = 0 \text{ and } \sigma_{KL} = \sigma \quad (2)$$

In equation 1, σ represents the elasticity of substitution between capital and labor, and θ and $1-\theta$ are the factor cost share of capital and labor, respectively. Equation (1) also serves as the basis for a class of production functions with variable elasticity of factor substitutions (VES) that assumes $\beta_2 \neq 0$. This makes σ_{KL} a function of the capital–labor ratio. Moreover, studies show that factor income shares can also vary with the capital–labor ratio. Karagiannis, Palivos, and Papageorgiou (2005) consider a VES framework (Equation 3) assuming σ_{KL} as a linear function of the capital–labor ratio (Revankar 1971), and provide empirical support to it. Taken together, it points to the advantages of a VES framework in addressing movements in the labor income share with varying levels of capital per unit of labor.

$$Y = AK^{\alpha\left(1-\theta\frac{\sigma-1}{\sigma}\right)}\left[\left(\frac{1-2\sigma}{\sigma}\right)K^{\frac{\sigma-1}{\sigma}} + L\right]^{\alpha\left(\theta\frac{\sigma-1}{\sigma}\right)};\ \beta_2 \neq 0 \text{ and } \sigma_{KL} = 1 + \frac{1-2\sigma}{\sigma-\theta(1-\sigma)}\frac{K}{L} \quad (3)$$

[1] A standard form of a CD production function can be written as $Y = AK^{\alpha}L^{1-\alpha}$, where the elasticity of substitution (σ_{KL}) between capital and labor is equal to unity.

However, most of the studies (Bentolila and Saint-Paul 2003; Elsby, Hobijn and Sahin 2013) that derive a theoretical relationship between factor income shares and σ_{KL} rely on a CES production function. In a CES production function (equation 2), assuming constant returns to scale and perfectly competitive factor markets, there is a stable relationship between the labor income share, the elasticity of substitution (σ) and capital–output ratio. Under these assumptions and using the aggregate production function (2), the labor income share can be derived as

$$L_S = \frac{(L)^{\frac{\sigma-1}{\sigma}}}{(K)^{\frac{\sigma-1}{\sigma}} + (L)^{\frac{\sigma-1}{\sigma}}} \tag{4}$$

and the capital-output ratio as

$$k = \left[\frac{(K)^{\frac{\sigma-1}{\sigma}}}{(K)^{\frac{\sigma-1}{\sigma}} + (L)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{5}$$

Combining (4) and (5), I get

$$L_S = 1 - (k)^{\frac{\sigma-1}{\sigma}}. \tag{6}$$

The expression for the labor income share in equation (6) is known as the “SK” schedule (Bentolila and Saint-Paul 2003), which shows a functional relationship between the labor income share, σ, and capital–output ratio. When σ > 1, i.e., labor and capital are gross substitutes, availability of more capital per unit of labor reduces the labor income share as the capital price goes down. This is known as “accumulation view.” Similarly, when σ < 1, i.e., labor and capital are gross complements, a higher *k* increases the labor income share.

Both Piketty (2014), and Karabarbounis and Neiman (2014), estimate $\sigma_{KL}$to be greater than unity. A CES production function assumes (i) the existence of the relationship between value added per unit of labor and the wage rate independent of the changes in the stock of capital (i.e., $\beta_2 = 0$) and (ii) the elasticity of substitution between factor inputs as a constant (but not unity) along the isoquant. Both assumptions appear unrealistic in the presence of an upward trend in the capital–labor ratio as documented by many studies (Acemoglu and Guerrieri 2008).

Nonetheless, it remains an empirical question whether a VES is a more realistic model compared to CES to study changes in the factor income shares. The recent growth in the labor income share literature mostly relies on the CES model to derive the relationship between the elasticity of factor substitution and the labor income share. The literature that provides empirical validity to the usefulness of VES production technology predates the recent growth in the labor income share research. While most of the studies (Sato and Hoffman 1968; Diwan 1970; Kazi 1980; Meyer and Kadiwala 1974; and Revankar 1971) reject CD and CES model specifications in favor of the VES model, Lovell (1973), Tsang and Yeung (1976) and Zellner and Ryu (1998) provide evidence that in certain sectors the CES model provides a better fit to the data compared to VES. Since these studies use various estimation strategies methods and different sets of data (both cross-sectional and time-series) at different levels of aggregation (from industries within a country to cross-country countries using aggregate data), it becomes difficult to ascertain a definite answer.

In this paper, I argue that the VES model specification is preferred to the CES to explain the movements in the labor income share. I work with a production function where σ_{KL} is a non-linear function of $\frac{K}{L}$ ($\beta_2 \neq 0$ in equation 1). I build on the variable elasticity of factor substitution production framework originally developed by Lu and Fletcher (1968), which allows σ_{KL} to vary with the factor shares as the availability of capital per unit of labor changes. I perform two empirical exercises. First, I test whether $\beta_2 = 0$ using a panel data on 108 Japanese industries throughout almost 40 years (1970 – 2012).[2] Second, I derive σ_{KL} for the industries at a disaggregated level of classification.

The empirical findings suggest a consistent and statistically significant role that the capital–labor ratio plays in explaining the variation in output per worker over time and across sectors. This suggests a variable elasticity of substitution between capital and labor as a function of the capital per unit of labor. Almost 40% of the industries show a statistically significant variation in $\hat{\sigma}_{KL}$ around its average value, which supports the role of capital per unit of labor in the movement of the elasticity of factor substitution over time. Also, 15 out of 23 industries have the average estimated elasticity of substitution between capital and labor greater than unity. This indicates that capital and labor are gross substitutes in most of the sectors. The substitutability between capital and labor is more prevalent among service industries compared to those in manufacturing or agriculture. Overall, the findings suggest that the CES model may not always be an ideal choice to examine the drivers of the labor income share. Movements in the labor income share have direct bearings on income distribution and input use, and the findings in this paper suggest more careful attention to model selection in order.

This paper is directly related to the recent debate on the role of the elasticity of factor substitution behind the secular decline in the labor income share. Using the CES production framework, Piketty (2014) and Karabarbounis and Neiman (2014) estimate the values of elasticity of substitution between capital and labor to be greater than unity. However, many studies find an estimate of σ to be less than one (Leon-Ledesma, McAdam and Willman 2015; Oberfield and Raval 2014; Chirinko and Mallick 2017). These findings point to an apparent puzzle. I mention two recent studies that attempt to resolve this puzzle. Grossman et al. (2017) show that a decline in the labor income share is feasible with $\sigma < 1$ if there is a slowdown of labor productivity growth. Paul (2018), in another study, drawing insights from the literature on differential capital–skill substitutability (Krusell et al., 2000) and the estimation of different elasticity of substitution parameters using the Morishima elasticity of substitution, shows that it is possible to have a decline in the labor income share resulting from a fall in the relative price of capital when $\sigma_{Agg} < 1$. This study provides another alternative to address this puzzle using the VES framework.

This paper is also related to the literature on the endogenous elasticity of factor substitution. Miyagawa and Papageorgiou (2007) build a static factor-endowment model where σ_{KL} in each period is endogenously determined by the existing endowments of capital and labor and their equilibrium inter-sectoral allocation. Duffy and Papageorgiou (2000), in another paper, show that σ_{KL} increases as the economy grows. This line of literature does not assume that capital per unit of labor and σ_{KL} are related based on any functional form. Rather, such a relationship is determined by the market equilibrium conditions of a growing multi-sectoral economy. This paper is directly linked to the endogenous elasticity of factor substitution but unlike Miyagawa and Papageorgiou (2007), it assumes a non-linear relationship between σ_{KL} and capital per unit of labor.

[2] The Japanese Industrial Productivity (JIP) database.

The rest of the paper is organized as follows. In section II, I provide a brief discussion of the evolution of VES production functions since the 1960s. I then discuss a VES framework that I empirically test in this paper. Section III provides empirical evidence using the Japanese Industrial productivity (JIP) database, which is followed by a concluding remark.

2. A BRIEF HISTORY OF THE EVOLUTION OF THE VES PRODUCTION FUNCTIONS

A production function portrays the techniques of how inputs are used to produce the output. It shows both the technical efficiency and allocative efficiency of the inputs. Production function has been an important tool of economic analysis in the neoclassical tradition. Economists have typically assumed that the factor inputs are technically efficient, and as a result, production functions in economic analysis focus on the allocative efficiency of the factor inputs. Philip Wicksteed (1894) was the first economist to algebraically formulate the relationship between output and n inputs as $Y = f(x_1, x_2, .., x_n)$, while some sources suggest that Johann von Thünen first formulated it in the 1840s (Humphrey 1997). A standard CD (Cobb-Douglas, 1928) production function (equation 4) demonstrates allocative efficiency from changes in the input uses and how it affects the output, which is otherwise assumed to be technically efficient.

$$Y = AK^{\alpha}L^{1-\alpha}. \tag{7}$$

In a CD production function (equation 7), the elasticity of substitution (σ_{KL}) between capital and labor is equal to unity. Unitary σ_{KL} supports Kaldor's stylized facts on constant factor income shares, and for several years it has been considered as a "deep" parameter. The growing dissatisfaction with the Cobb–Douglas production function led to the invention of the CES production function. The CES production function was derived almost 33 years after the formulation of the CD production function based on the goodness of fit of the empirical relationship as shown in equation (1) with $\beta_2 = 0$ (Arrow et al. 1961). The CES production allows for non-unitary values of σ_{KL}. However, it does not allow σ_{KL} to vary with changes in the capital per unit of a worker. In other words, σ_{KL} remains constant across the isoquants independent of the size of the output and inputs in the production function.

Since the formulation of the CES production function, several attempts[3] have been made to accommodate the variability of σ_{KL} in the production function. Mukerji (1963) generalized the CES production function based on constant ratios of σ_{KL}. Revankar (1967) developed a generalized CES production function with variable returns to scale and elasticity of substitution. Revankar's VES, or the generalized CES production function (equation 3), does not contain the Leontief production function but shows the Harrod–Domar fixed coefficient model, the linear production function and the CD production function as its special cases. In this model, σ_{KL} varies linearly with the capital per unit of labor. However, it does not allow the value of σ_{KL} to cross over from one side of the unity to the other in the relevant range of the capital–labor ratio. Bruno (1968)

[3] Bruno (1962); Brown and Cani (1963); Mukerji (1963); Nerlove (1963); Ringstad (1967); Revankar (1967); Lu and Fletcher (1968); Sato and Hoffman (1968); Revankar (1971) and Kadiyala (1972), among others. Please see Mishra (2007) for a comprehensive analysis of the evolution of production functions in economic analysis.

formulated constant marginal share (CMS) production function (equation 8), where labor productivity increases with capital per unit of labor, but at a decreasing rate (θ).

$$\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha} - \theta. \tag{8}$$

As a result, in a CMS production function, σ_{KL} is less likely to be less than unity (equation 9).

$$\sigma_{KL} = 1 - \frac{\theta\alpha}{(1-\alpha)}\frac{L}{Y}. \tag{9}$$

One year later, Lu and Fletcher (1968) developed a VES production function as a generalized function of CES that permits σ_{KL} to vary with the factor shares. Lu and Fletcher's (1968) VES model successfully overcame the problem of the monotonic relationship between σ_{KL} and the capital–labor ratio, which the Revankar (1967) model suffered from. They derived a VES model assuming minimum cost conditions, i.e., assuming a perfectly competitive labor and product market. To derive the production function, Lu and Fletcher (1968) used a log-linear form of the relationship between value added per unit of labor, a constant term (β_0), the wage rate (W), the capital–labor ratio ($\frac{K}{L}$) and an error term (ε) as shown in equation (1). Equation (10) contains a version of the Lu and Fletcher (1968) model, which establishes a direct relationship with equation (1). In this model, the labor input is multiplied by an additional factor, the capital per unit of labor.

$$Y = A\left[\theta K^{\frac{\sigma-1}{\sigma}} + \left\{\frac{1-\beta_1}{1-\beta_1-\beta_2}\left(\frac{K}{L}\right)^{\frac{-\beta_2}{\sigma}}\right\}(1-\theta)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{10}$$

If $\beta_2 = 0$, then it takes the form of a standard CES production function. This is consistent with equation (1), where $\beta_2 = 0$ suggests no role played by the changes in the capital per unit of labor to explain the variation in σ_{KL}. The production function in equation (10) has positive marginal products of both input factors, a downward sloping marginal productivity curve over the relevant ranges of inputs, shows constant returns to scale (homogeneous of degree 1) and shows a variable elasticity of factor substitution (equation 11).

$$\sigma_{KL} = \frac{\beta_1}{1-\beta_2\left(1+\frac{1}{\frac{1-\theta}{\theta}\left[\left(\frac{K}{L}\right)^{\frac{\beta_1+\beta_2-1}{\beta_1}} - \frac{\beta_2}{\beta_1+\beta_2-1}\right]}\right)}. \tag{11}$$

Following the Hicksian elasticity of factor substitution between capital and labor,[4] the production technology in equation (10) suggests that the marginal rate of technical substitution (MRS) becomes a function of the capital–labor ratio. In other words, moving

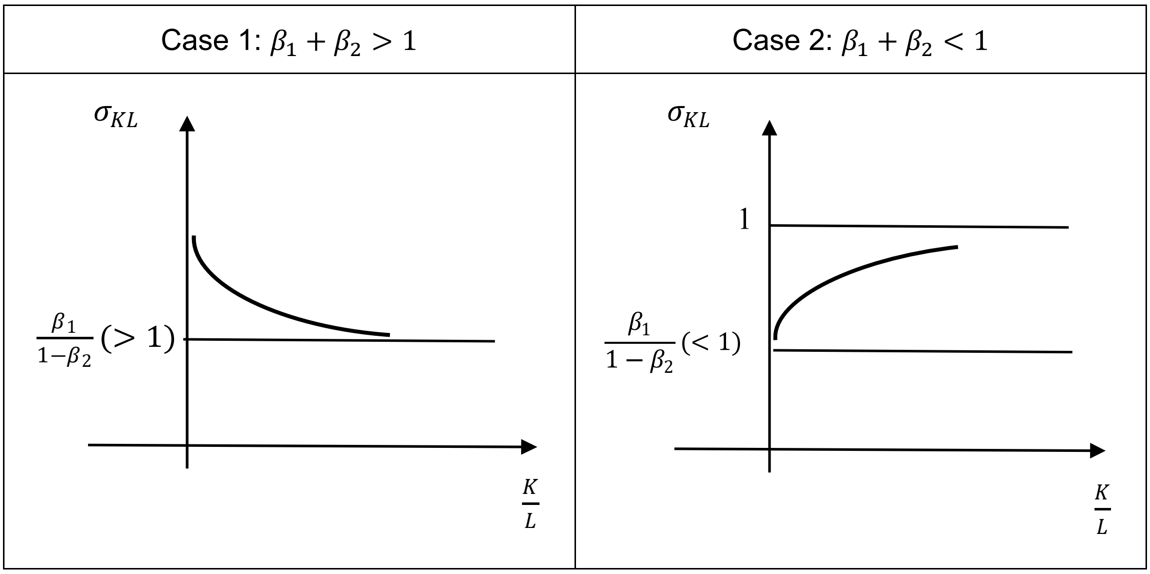
[4] The Hicksian elasticity of factor substitution is defined as $\sigma_{KL} = -\frac{\frac{d\left(\frac{K}{L}\right)}{\frac{K}{L}}}{\frac{d(MRS)}{MRS}}$. MRS stands for the marginal rate of technical substitution between capital and labor.

along the isoquant, σ_{KL}varies with the $\frac{K}{L}$. Equation 12 shows σ_{KL} as a non-linear explicit function of $\frac{K}{L}$. Taking derivatives of σ_{KL} with respect to $\frac{K}{L}$, I get

$$\frac{d(\sigma_{KL})}{d(\frac{K}{L})} = -\frac{\frac{\beta_1+\beta_2-1}{\beta_1}(1-\theta)\beta_1\beta_2\left(\frac{K}{L}\right)^{\frac{\beta_1+\beta_2-1}{\beta_1}-3}\left(\frac{w}{r}\right)^2}{\theta\left[1-\beta_2\left(1+\frac{wL}{rK}\right)\right]^2}. \qquad (12)$$

Thus, if $\frac{\beta_1+\beta_2-1}{\beta_1} \gtreqless 0$, then I have $\frac{d(\sigma_{KL})}{d(\frac{K}{L})} \lesseqgtr 0$. Figure 1 plots the relationship between σ_{KL} and the $\frac{K}{L}$. In case 1, when $\beta_1 + \beta_2 > 1$, then σ_{KL} declines with an increasing $\frac{K}{L}$, and asymptotically approaches to $\frac{\beta_1}{1-\beta_2} > 1$. In case 2, when $\beta_1 + \beta_2 < 1$, the value of σ_{KL} increases from $\frac{\beta_1}{1-\beta_2} < 1$ to unity, as $\frac{K}{L}$ increases from 0 to infinity.

Figure 1: Relationship between $\sigma_{KL}$and Ratio between Capital and Labor



Source: Author.

3. EMPIRICAL ANALYSIS

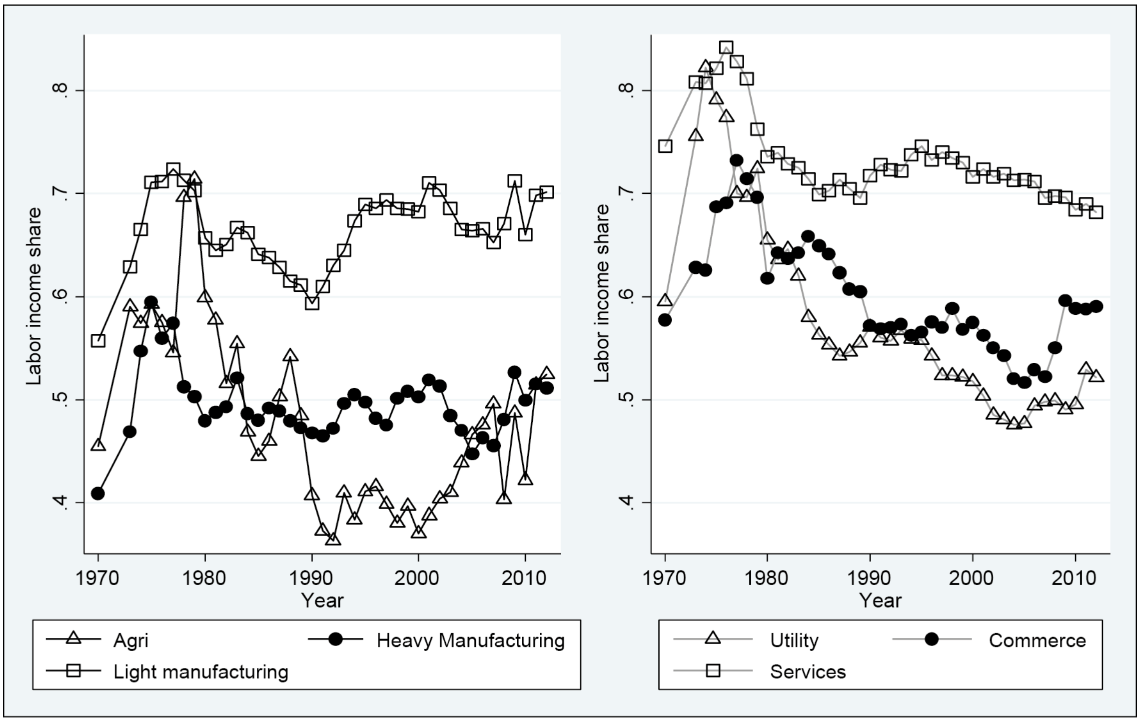
3.1 Data and Descriptive Statistics

I used the Japan Industrial Productivity (JIP) and the Regional Japan Industrial Productivity (R-JIP) databases compiled by RIETI (Research Institute of Economy, Trade, and Industry) and Hitotsubashi University, Tokyo.[5] The latest round of the JIP database (2015) covers 108 industries for the period 1970–2012. Following Fukao and Perugini (2018), I constructed the labor income share by sector (industry) as the ratio of nominal total labor compensation to nominal value added (at current prices). Since nominal total labor compensation includes all types of remuneration, such as employee

5 See https://www.rieti.go.jp/en/database/JIP2015/#01. For a detailed account of the JIP database, see Fukao et al. (2015). JIP sectors can be easily translated into international industry classifications such as ISIC and KLEMS.

compensation and mixed income (i.e., for labor supplied by self-employed and family workers), it automatically adjusts for labor compensation of nonworkers (employees). This makes the labor income share measure less susceptible to measurement errors as highlighted by many researchers (Gollin 2002; Guerriero 2012). Also, I used the Regional-Level Japan Industrial Productivity (R-JIP) database,[6] which consists of 23 sectors (agriculture, mining, food, textiles, pulp, chemicals, petroleum, nonmetallic minerals, primary metals, fabricated metals, machinery, electrical machinery, transport equipment, precision instruments, other manufacturing, construction, utilities (electricity, gas and water supply), wholesale and retail trade, finance and insurance, real estate, transport and communication, private services and government services). I merged this data set into the JIP database, mainly to facilitate the creation of the classification of broad sectors.

Figure 2: Sectoral Labor Income Share in Japan, 1970–2012



Note: Author's calculation based on the Japan Industrial Productivity (JIP) database https://www.rieti.go.jp/en/database/JIP2015/#01, and Regional-Level Japan Industrial Productivity (R-JIP) database, http://www.rieti.go.jp/en/database/r-jip.html. The latter data set consists of 23 sectors. I divided them into six broad categories.

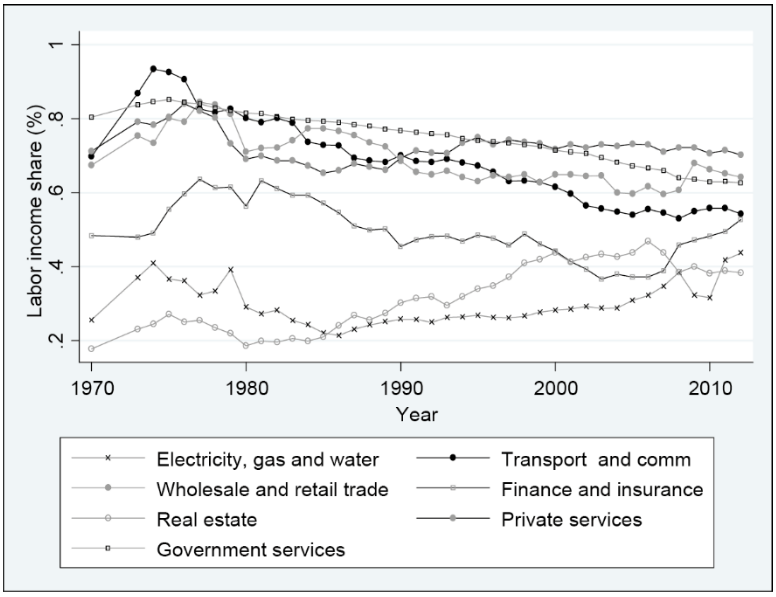
I divided 108 industries into six broad categories of sectors. *Agri* consists of agriculture, forestry, and fisheries. *Heavy manufacturing* comprises mining, chemicals, petroleum, fabricated metals, machinery, construction and electrical machinery. *Light manufacturing* consists of food, textiles, pulp, nonmetallic minerals, primary metals, transport equipment, precision instruments and other manufacturing. *Utilities* include electricity, gas and water supply, transport and communication. *Commerce* consists of wholesale and retail trade, finance and insurance, and real estate. I included both private services and government services in *Services*. Figure 2 shows labor income share trends for these six broad sectors. The labor income shares remained almost constant in heavy manufacturing and light manufacturing, whereas the other sectors—

[6] http://www.rieti.go.jp/en/database/r-jip.html (It should be noted that data are missing for Okinawa for the period 1955 to 1970.)

utility, commerce, and services—showed downward trends in the period from 1970 to 2012. The labor income share in the agriculture sector started rising since the early 1990s after a significant drop in the 1980s. The industries under the utilities sector, on average, show the steepest fall in the labor income share.

To get a closer look at the drivers of the downward trend in utilities, commerce, and services, Figure 3 plots the sectoral labor income share trends at a more disaggregated level of sectors. I now considered seven sectors: (i) electricity, gas and water supply; (ii) transport and communication; (iii) wholesale and retail trade; (iv) finance and insurance; (v) real estate; (vi) private services and (vii) government services. Labor income shares in sectors like electricity, gas and water, and real estate do not show any downward trend. Labor's share of income in private services also did not fall significantly since the late nineties. The rest of the sectors contribute to a falling labor income share in utilities, commerce and services.

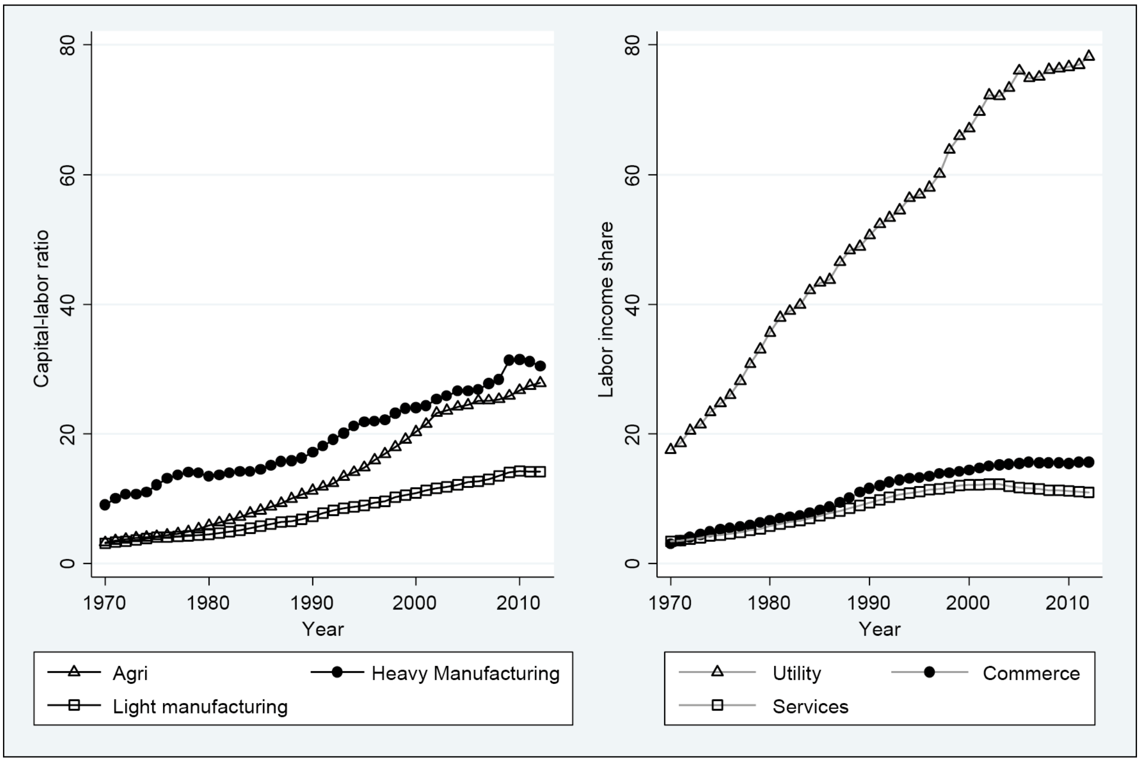
Figure 3: Sectoral Labor Income Share in Utility Commerce and Services (disaggregated level): 1970–2012



Note: Author's calculation based on the Japan Industrial Productivity (JIP) database. https://www.rieti.go.jp/en/database/JIP2015/#01, and Regional-Level Japan Industrial Productivity (R-JIP).

Figure 4 shows the changes in the capital per unit of labor for six broad categories of sectors. I measured capital by using both IT and non-IT capital stock (valued at 2000 prices), and labor was measured by the number of employed persons in each sector. The capital–labor ratio increased in the period from 1970 to 2012 across all sectors. However, industries in the utility sector, on average, show the most rapid increase in the capital per unit of labor between 1970 and 2010. Overall, steady growth in the capital–labor ratio across Japanese industries supports the arguments for capital deepening (Acemoglu and Guerrieri 2008), capital-intensive technological change (Acemoglu 2002) and capital–skill complementarity (Krusell et al. 2000). This consistent upward trend of capital–labor ratio also makes a case for variable elasticity of factor substitution stronger. In the next section, I provide empirical support to this assertion. Appendix 2 shows labor income share at a more disaggregated level for light and heavy manufacturing industries.

Figure 4: Capital-Labor Ratio by Broad Sectors, 1970–2012



Note: Author's calculation based on the Japan Industrial Productivity (JIP) database. https://www.rieti.go.jp/en/database/JIP2015/#01, and Regional-Level Japan Industrial Productivity (R-JIP) database, http://www.rieti.go.jp/en/database/r-jip.html. The latter data set consists of 23 sectors. I divided them into six broad categories.

3.2 Empirical Model

The main purpose of the empirical analysis is two-fold. First, it estimates the relationship between the capital–labor ratio and output per worker. Using equation (1), I tested the null hypothesis if $\beta_2 = 0$. I rewrote equation (1) in the form of a panel regression model (Equation 10), where i indicates disaggregated categories of sectors (108 in total)[7] and t stands for a year (from 1970 to 2012).

$$log\left(\frac{Y}{L}\right)_{it} = \beta_0 + \beta_1 logW_{it} + \beta_2 log\left(\frac{K}{L}\right)_{it} + \varepsilon_{it}\ . \tag{13}$$

Second, I used the estimated coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$ from equation 10 and calculated the elasticity of substitution between capital and labor ($\hat{\sigma}_{KL}$) using Equation 11 at the disaggregated sectoral level.

$$\hat{\sigma}_{KL} = \frac{\hat{\beta}_1}{1-\hat{\beta}_2\left(1+\frac{1}{\frac{1-\theta}{\theta}\left[\left(\frac{K}{L}\right)^{\frac{\hat{\beta}_1+\hat{\beta}_2-1}{\hat{\beta}_1}}-\frac{\hat{\beta}_2}{\hat{\beta}_1+\hat{\beta}_2-1}\right]}\right)}. \tag{14}$$

[7] See Appendix 1 for mapping between these 108 sectors and six broad categories of sectors.

3.3 Empirical Outcomes

Table 1 shows the outcomes of the fixed-effect panel estimation of the regression model in equation 10. The first column includes the total sample (including 108 sectors) in the estimation, and the estimated coefficients of both log wages and log capital–labor ratio are positive and statistically significant at the 1% level. The F-test outcome rejects the null hypothesis that $\hat{\beta}_2 = 0$, at 1% level statistical significance. Columns 2–7 show outcomes at the broad sectoral level. The estimated coefficients of log wage show the same sign and statistical significance across all model specifications. However, the capital–labor ratio is not statistically significant for industries under commerce.

And consequently, the F-test outcome indicates that the capital–labor ratio has no statistically significant relationship with output per worker for industries in commerce. Overall, the estimated coefficients suggest a consistent and statistically significant role that capital–labor ratio plays in explaining the variation in output per worker over time and across sectors. This also suggests a variable elasticity of substitution between capital and labor as shown in equation 11.

Table 1: Regression Outcomes on Output per Worker (1970–2012)

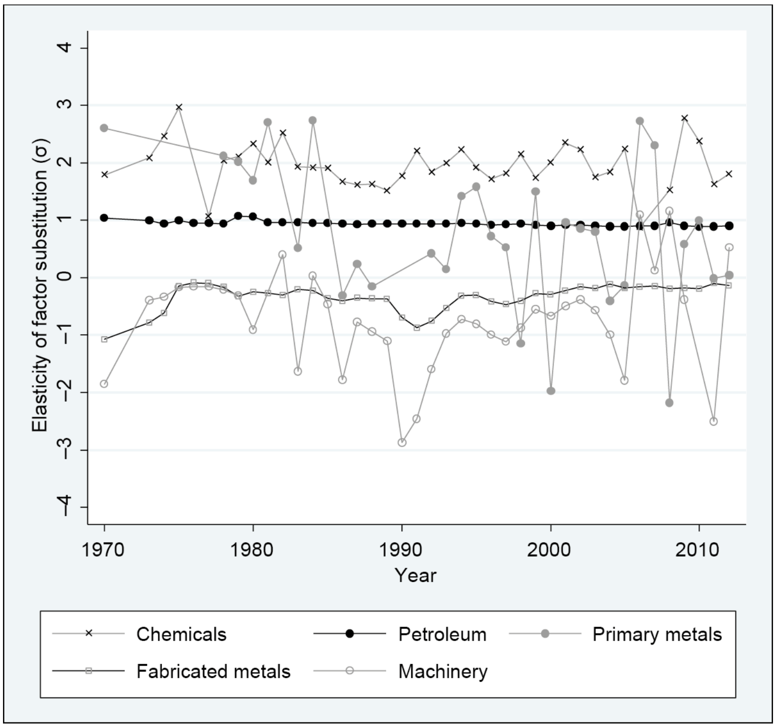
	1	2	3	4	5	6	7
	All Sectors	**Agri**	**Heavy Manufacturing**	**Light Manufacturing**	**Utilities**	**Commerce**	**Services**
Log wage	0.903***	0.486***	0.844***	0.866***	1.007***	1.011***	1.013***
Log (K/L)	0.099***	0.365***	0.134***	0.077***	0.242***	0.021	–0.049***
Constant	0.435***	–0.261***	0.666***	0.519***	–0.224**	0.579***	0.348***
N	4,345	246	778	1,517	492	205	1,107
R^2	0.813	0.798	0.763	0.909	0.813	0.825	0.803
F- test ($\hat{\beta}_2 = 0$)	145.24***	284.18***	25.29***	70.23***	52.30***	0.13	48.61***

Note: *** significance at 1%, ** significance at 5% and * significance at 10%.

Moving on, I next calculated the elasticity of substitution between capital and labor for 23 R-JIP sectors mapped into the broad categories in the following way: *Agriculture* comprises of agriculture, forestry and fisheries. *Heavy manufacturing* includes mining, chemicals, petroleum, fabricated metals, machinery, construction and electrical machinery. *Light manufacturing* consists of food, textiles, pulp, nonmetallic minerals, primary metals, transport equipment, precision instruments and other manufacturing. *Utilities* includes electricity, gas, water supply, transport and communication. *Commerce* consists of wholesale and retail trade, finance and insurance, and real estate. And finally, service includes both private services and government services.

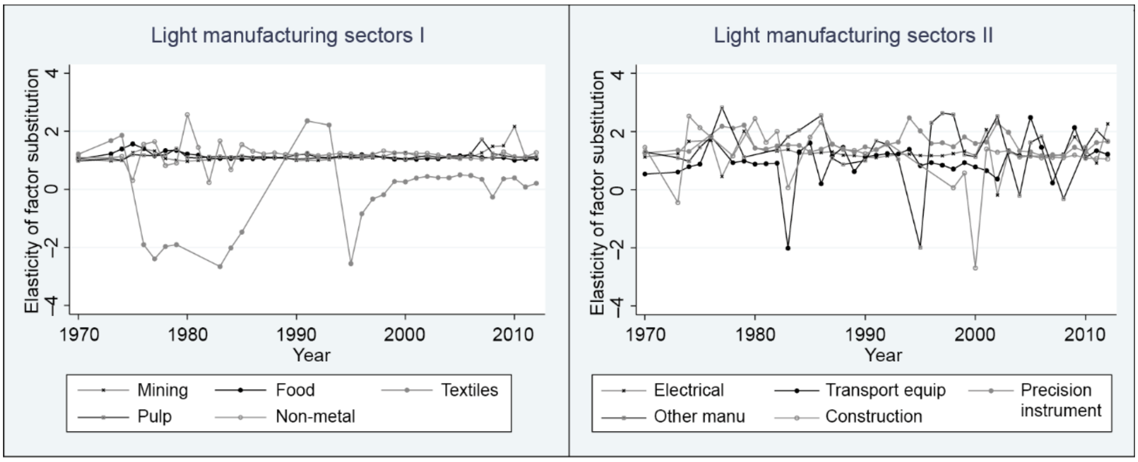
Figure 5 plots the elasticity of substitution between capital and labor for five heavy manufacturing sectors. Except for petroleum, the other four sectors (chemicals, primary metals, fabricated metals and machinery) show considerable variation in the measure of substitution elasticity. The petroleum sector shows unitary elasticity between capital and labor that is constant over time. For chemical industries, the value of the substitution elasticity is greater than one, whereas the same for machinery and fabricated metal is consistently less than unity with considerable variation. The sectors in the primary metal show the most frequent oscillation, with the estimates of the substitution parameter ranging between –2 and 3. Thus, the relationship between factor inputs not only varies over time but it also exhibits mixed trends of complementarity and substitutability over time.

Figure 5: Elasticity of Substitution between Capital and Labor in Heavy Manufacturing Sectors



Note: Author's calculation.

Figure 6: Elasticity of Substitution between Capital and Labor in Light Manufacturing Sectors

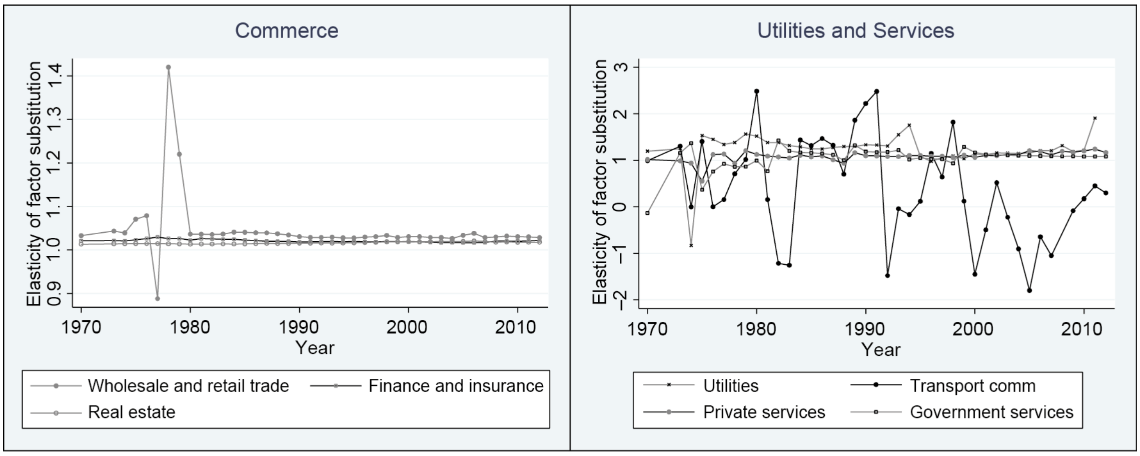


Note: Author's calculation.

Figure 6 plots the elasticity of substitution between capital and labor for the light manufacturing sectors. The left-hand panel of Figure 6 exhibits substitution elasticity for mining, food, textiles, pulp and non-metal industries. The trends of substitution elasticity over time suggest it shows constant trends for most of the sectors except textiles. The right-hand panel of Figure 6 shows the same for other light-manufacturing sectors. The trends based on the estimated elasticity of substitution for sectors such as electrical, transportation equipment, precision instrument, other manufacturing and construction show greater variation over the period from 1970–2012, and the average values of the values of $\hat{\sigma}_{KL}$ tend to vary between 1 and 2. I return to this point with a more elaborated discussion using Table 2.

Among the three sectors in commerce, the variation in the elasticity of substitution for the wholesale and retail trade sector is far greater than finance and insurance and real estate (the left-hand panel of Figure 7). Among the sectors in utilities and services, only transport and communications show significant variation in $\hat{\sigma}_{KL}$. The average value of $\hat{\sigma}_{KL}$ in most of these sectors tends to be greater than unity.

Figure 7: Elasticity of Substitution between Capital and Labor in Commerce, Utilities and Services



Note: Author's calculation.

To conclude, I summed the statistical outcomes on labor income share, $\hat{\sigma}_{KL}$, and the nature of the substitutability between capital and labor across 23 broad Japanese sectors (R-JIP classification) for the period from 1970 to 2012. The third column in Table 2 shows the average labor income share over the period for each sector. As discussed earlier, the average share of labor's income is much lower in industries in agriculture and heavy manufacturing compared to the same in light industry, commerce and services. And, 15 out of 23 industries have the average estimated elasticity of substitution between capital and labor greater than unity (the fourth column, Table 2). Almost 40% of the industries show a statistically significant variation in $\hat{\sigma}_{KL}$ around its average value, which supports the role of capital per unit of labor in the movement of the elasticity of factor substitution over time.

The empirical findings also suggest that the relationship between capital and labor in most of the sectors is as substitutes. If the estimated value of the elasticity parameter is above unity in more than 60% of the years in the period between 1970 and 2012, then I consider capital and labor as gross substitutes. On the other hand, capital and labor are gross complements if $\hat{\sigma}_{KL} > 1$ in less than 40% of the times in the study period. Using this rule of thumb, only four industries, namely agriculture, textiles, primary metals and transport equipment, appear in the borderline case. There is complementarity between capital and labor in four industries: petroleum and coal products; fabricated metal; machinery and transport; and communication. The capital and labor are substitutes in the rest of the 15 industries. Substitutability between capital and labor, on average, is higher among the industries in services compared to manufacturing or agriculture.

Table 2: Substitutability of Sectoral $\hat{\sigma}_{KL}$ and Its Variability over Time

	R-JIP 23 Sectors	Average Labor Income Share	Mean of $\hat{\sigma}_{KL}$	t-Statistic (Mean/SD) of $\hat{\sigma}_{KL}$	Percentage of Years (1970–2012) $\hat{\sigma}_{KL} > 1$	The Nature of the Relationship between K and L
1	**Agriculture, Forestry and Fishery**	**0.48**	**0.54**	**0.54**	**41%**	**Borderline**
2	Mining	0.55	1.14	5.50*	85%	Substitutes
3	Food products and beverages	0.44	1.14	9.63*	100%	Substitutes
4	**Textiles**	**0.94**	**–0.15**	**–0.11**	**40%**	**Borderline**
5	Pulp, paper and paper products	0.52	1.13	9.69*	98%	Substitutes
6	Chemicals	0.37	1.96	4.87*	98%	Substitutes
7	Petroleum and coal products	0.08	0.94	22.86*	7%	Complements
8	Non-metallic mineral products	0.60	1.20	3.38*	88%	Substitutes
9	**Primary Metals**	**0.43**	**0.77**	**0.60**	**51%**	**Borderline**
10	Fabricated metal products	0.81	–0.33	–1.46	0%	Complements
11	Machinery	0.71	–0.73	–0.83	10%	Complements
12	Electrical machinery, equipment and supplies	0.64	1.27	3.20*	93%	Substitutes
13	**Transport equipment**	**0.57**	**0.98**	**1.47**	**49%**	**Borderline**
14	Precision instruments	0.72	1.58	4.82*	100%	Substitutes
15	Other manufacturing	0.73	1.42	1.47	85%	Substitutes
16	Construction	0.78	1.17	1.23	88%	Substitutes
17	Electricity, gas and water supply	0.30	1.24	3.22*	95%	Substitutes
18	Wholesale and retail trade	0.69	1.05	14.64*	98%	Substitutes
19	Finance and insurance	0.50	1.02	328.5*	100%	Substitutes
20	Real estate	0.32	1.02	455.4*	100%	Substitutes
21	Transport and communication	0.69	0.39	0.35	34%	Complements
22	Service activities and producers of private non-profit services to households	0.72	1.09	9.80*	88%	Substitutes
23	Producers of government services	0.75	1.05	4.03*	78%	Substitutes

Note: Author's calculation.

4. CONCLUSION

The crucial role of σ in analyzing the factor income shares has been noted since the seminal work of Hicks (1932) and Robinson (1953). The CES production function has predominantly been used to derive the relationship between σ and the labor income share. Assuming constant returns to scale and perfectly competitive factor markets, there is a stable relationship between factor income shares and σ. And, the constancy of the elasticity of factor substitution makes its role as a driver of the labor income share exogenous.

This paper suggests that more careful attention must be paid in the modeling choice. The empirical findings using the Japan Industrial Productivity database at the disaggregated level of industry classification suggest a consistent and statistically significant role that capital–labor ratio plays in explaining the variation in output per worker over time and across sectors. This validates the existence of a variable elasticity of substitution between capital and labor. Almost 40% of the industries show a statistically significant variation in $\hat{\sigma}_{KL}$ around its average value. Also, capital and labor are gross substitutes in most of the sectors. The substitutability between capital and labor is more prevalent among the industries in services compared to that in manufacturing.

The topic of income distribution has always been at the center of economic policymaking, and the recent decline in the labor income share has generated concerns among researchers and policymakers, alike. And, the puzzling role of σ in explaining the movements in the labor income share as highlighted by some recent studies (Grossman et al. 2017; Paul 2018) adds to the misery. This study provides a novel mechanism to resolve this puzzle that allows σ to vary over time. A framework with both σ and the labor income share varying over time lead to endogeneity issues, and credible measures must be taken to address it. I leave this task for future studies.

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APPENDIX 1

Broad Categories	JIP Sectors	JIP Code
1	Rice, wheat production	1
	Miscellaneous crop farming	2
	Livestock and sericulture farming	3
	Agriculture services	4
	Forestry	5
	Fisheries	6
2	Chemical fertilizers	23
	Basic inorganic chemicals	24
	Basic organic chemicals	25
	Organic chemicals	26
	Chemical fibers	27
	Miscellaneous chemical products	28
	Pharmaceutical products	29
	Petroleum products	30
	Coal products	31
	Pig iron and crude steel	36
	Miscellaneous iron and steel	37
	Smelting and refining of non-ferrous metals	38
	Non-ferrous metal products	39
	Fabricated constructional and architectural metal products	40
	Miscellaneous fabricated metal products	41
	General industry machinery	42
	Special industry machinery	43
	Miscellaneous machinery	44
	Office and service industry machines	45
3	Mining	7
	Livestock products	8
	Seafood products	9
	Flour and grain mill products	10
	Miscellaneous foods and related products	11
	Prepared animal foods and organic fertilizers	12
	Beverages	13
	Tobacco	14
	Textile products	15
	Lumber and wood products	16
	Furniture and fixtures	17
	Pulp, paper, and coated and glazed paper	18
	Paper products	19

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Appendix 1 *table continued*

Broad Categories	JIP Sectors	JIP Code
	Printing, plate making for printing and bookbinding	20
	Leather and leather products	21
	Rubber products	22
	Glass and its products	32
	Cement and its products	33
	Pottery	34
	Miscellaneous ceramic, stone and clay products	35
	Electrical generating, transmission, distribution and industrial apparatus	46
	Household electric appliances	47
	Electronic data processing machines, digital and analog computer equipment and accessories	48
	Communication equipment	49
	Electronic equipment and electric measuring instruments	50
	Semiconductor devices and integrated circuits	51
	Electronic parts	52
	Miscellaneous electrical machinery equipment	53
	Motor vehicles	54
	Motor vehicle parts and accessories	55
	Other transportation equipment	56
	Precision machinery and equipment	57
	Plastic products	58
	Miscellaneous manufacturing industries	59
	Construction	60
	Civil engineering	61
	Publishing	92
4	Electricity	62
	Gas, heat supply	63
	Waterworks	64
	Water supply for industrial use	65
	Waste disposal	66
	Railway	73
	Road transportation	74
	Water transportation	75
	Air transportation	76
	Other transportation and packing	77
	Telegraph and telephone	78
	Mail	79

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Appendix 1 *table continued*

Broad Categories	JIP Sectors	JIP Code
5	Wholesale	67
	Retail	68
	Finance	69
	Insurance	70
	Real estate	71
6	Education (private and non-profit)	80
	Research (private)	81
	Medical (private)	82
	Hygiene (private and non-profit)	83
	Other public services	84
	Advertising	85
	Rental of office equipment and goods	86
	Automobile maintenance services	87
	Other services for businesses	88
	Entertainment	89
	Broadcasting	90
	Information services and internet-based services	91
	Video picture, sound information, character information production and distribution	93
	Eating and drinking places	94
	Accommodation	95
	Laundry, beauty and bath services	96
	Other services for individuals	97
	Education (public)	98
	Research (public)	99
	Medical (public)	100
	Hygiene (public)	101
	Social insurance and social welfare (public)	102
	Public administration	103
	Medical (non-profit)	104
	Social insurance and social welfare (non-profit)	105
	Research (non-profit)	106
	Other (non-profit)	107

APPENDIX 2: SECTORAL LABOR INCOME SHARES