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Fiscal Policy in a Currency Union at the Zero Lower Bound *

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Abstract

When monetary policy is constrained by the zero lower bound, fiscal policy can be used to achieve macro stabilization objectives. At the same time, fiscal policy is also a key policy variable within a single currency area that allow policy-makers to respond to regional demand asymmetries. How do these two uses of fiscal policy interact with one another? Is there an inherent conflict between the two objectives? How do the answers to these questions depend on the degree of fiscal space available to different members of the currency area? This paper constructs a two-country New Keynesian model of a currency union to address these questions. We find that the answers depend sensitively on the underlying internal structure of the currency union, notably the degree of trade openness between the members of the union.

Keywords:Liquidity Trap, Monetary Policy, Fiscal Policy, International Spillovers

JEL: E2, E5, E6

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1 Introduction

Over the last decade, developed economies have been operating with very low interest rates, effectively at the zero bound. This has severely constrained the use of monetary policy in these countries. Constraints on the use of monetary stabilization raises the possibility of alternative stabilization instruments, including fiscal policy. Fiscal expenditure can be used to stimulate inefficiently low demand, when aggregate demand conditions are sufficiently severe that monetary policy rates are unable to fall further (see Woodford, 2010; Christiano, Eichenbaum and Rebelo, 2011). But optimal fiscal stabilization at the zero bound may be complicated in a currency union, because different regions within the union may be subject to different degrees of severity in macro shocks. In fact, it is well recognized that this problem is not simply a facet of the zero bound environment. In a group of economies jointly using a single currency, monetary policy will always be in some way constrained from addressing regional disparities in aggregate demand. The literature has addressed these question, showing that optimal regional fiscal policy may then be targeted toward locations with low demand (see Beetsma and Jensen, 2005; and Gali and Monacelli, 2008).

The different roles that fiscal policy might play in a currency union that is further constrained by the zero bound sets up the possibility of a conflict in the uses of fiscal policy. Consider the post global financial crisis European economy. The persistent disinflation experienced by the Eurozone was highly concentrated by region. More extreme downturns were observed in the peripheral countries than in the core countries such as Germany. In such a context, it is relevant to ask to what degree it is optimal to apply expansionary fiscal policy across all regions. Conversely, if fiscal demand should shift heavily enough toward the more badly affected regions, then in order to achieve a balanced degree of aggregate demand across the union, it may be potentially optimal to contract spending in the least affected regions.

We will examine this question in the context of a two-region New Keynesian model of a currency union. A key element of the model is that each region has a bias in consumption toward home-produced goods. Under home bias, a disinflationary demand shock in one region will be concentrated on goods in that region and will spread only partially to the other region. But if it is sufficiently severe, even a regional shock could move the aggregate economy of the union to the effective lower bound on interest rates.

Our approach to dealing with these trade-offs is to identify the optimal aggregate and relative composition of fiscal policy within the model. Optimality is defined using an approximation for the social welfare function of the currency union [1]. We concentrate on examining the social welfare maximizing optimal policies of the regions of the currency union, assuming that the union members can cooperate on fiscal spending policies, and where policies are chosen with discretion. Broadly speaking, we find that optimal policy in the most affected region should be expansionary. However, the optimal fiscal response of the partner region depends on a variety of factors governing the nature of the equilibrium. Crucially for our investigation, it is not always the case the the partner region should follow an expansionary fiscal policy.

A key parameter governing our results is the degree of home bias in consumption. In a general sense, we can translate this parameter as that which governs the degree of internal trade openness *within* the union. The greater is home bias in the consumption basket, the smaller the gains from trade and the lower is the ratio of trade to GDP for each country. If domestic demand is very concentrated on domestic goods, then asymmetric shocks will result in greater regional business cycle disparities, leading to a greater need to concentrate spending in a particular region. Moreover, the spillovers of fiscal policy from one region to another are reduced accordingly, as home bias concentrates the effects of fiscal policy. Thus, fiscal policy will be less effective in addressing demand shocks in another region. A useful way to to clarify this relationship is to show the relationship between fiscal multipliers and home bias. This is what we illustrate below.

Conversely, if the union-wide economy economy is particularly vulnerable to the aggregate downturn and trade is highly integrated, then meeting aggregate optimal spending goals may require expansion across the union. Here is it not just home bias, but other structural parameters that are important. For example, if demand is sufficiently interest sensitive, then a relatively large fiscal expansion may be required. This will tend to require fiscal policy expansion in both regions. Another important factor is the duration of the underlying demand shocks which generate the downturn. If shocks are very persistent, this will also increase the likelihood of possible spillovers across regions.

As noted above, we characterize social welfare as a second order approximation to the utility of the residents of the currency union. Following other literature on the zero bound in macro models, we focus on demand shocks which tend to push inflation and the output gap in the same direction. In the aggregate union economy, there would be no direct trade-off

[1] This follows Cook and Devereux (2011a).

between stabilizing the two goals, given an unconstrained monetary policy. This is not true of fiscal policy in the aggregate, since using fiscal spending for stabilization purposes changes the optimal mix of private to public goods, which will create inefficiencies in the composition of goods in the economy. When we look at relative, within union differentials in the impact of shocks, monetary policy would be ineffectual, even absent the zero bound constraint, so fiscal policy response would be necessary in order to respond to differential responses of output gaps and inflation. But again, this would create additional compositional distortions. Thus, along both aggregate and relative dimensions in response to negative demand shocks, a fiscal spending policy obtains at best a second-best optimum.

While it has been well recognized that fiscal policy responses may be necessary for stablization within a currency union (e.g. Beetsma and Jensen, 2005), the additional dimension brought by the zero bound constraint adds a new element. An important principle in the analysis we follow below is that while the relative responses of fiscal policy to regionally concentrated shocks is independent of the zero bound constraint, the absolute response of each region depends critically upon whether the zero bound is binding or not. In practice, this means that in some cases, depending on the structural parameters and especially on the degree of home bias, the region less affected by the shock may need to contract fiscal policy in order to achieve efficient relative price adjustment, while in other cases, it may need to expand fiscal policy, in order to attain an optimal aggregate demand expansion.

We also consider optimal fiscal policy when some regions are fiscally constrained. We find that at the zero lower bound, aggregate fiscal policy will be expansionary under a wide variety of circumstances even when fiscal policy is constrained. Thus, when only one region can implement expansionary fiscal policy at the zero lower bound, that region will implement fiscal expansion regardless of whether that region is the most deeply affected. In our model (as in the continuous time model of Farhi and Werning, 2016), cross-country fiscal spillovers are positive at the zero lower bound, so a cooperative country can positively impact its demand constrained neighbor through its own fiscal expansion. This is in general not true in an environment of unconstrained monetary policy.

We follow a large open economy literature which studies fiscal policy in sticky price models at the zero lower bound. Fujiwara and Ueda (2013) identify fiscal multipliers in an open economy with flexible exchange rates. Cook and Devereux (2010, 2011a) examine optimal fisal policy at the zero lower bound in a two country model with flexible exchange rates, focusing on the importance of home bias. Hettig and Mueller (2015) examine the coordination of fiscal policy in a union of many small economies at the zero lower bound;

this paper examines only fiscal policy coordination.

Blanchard, Erceg and Linde (2016) asks similar questions as this paper. They examine the welfare gains from fiscal expansion at the zero lower bound in a two country sticky price model. They find welfare gains from fiscal expansions in both countries, whether jointly or separately. We build on this by characterizing optimal policy, though in a much simpler model.

2 A two country model

Consider a currency area made up of two regions. We assume that both of these regions have permanently committed to a single currency unit. In each country, households consume both private and government goods, and supply labor. Denote the countries as 'South' and 'North', with North variables denoted with an asterisk superscript. The population of each country is normalized to unity. Monopolistically competitive firms in each region produce differentiated goods with constant returns to scale technology. Regional governments produce government goods which are distributed uniformly across households within the region. Governments have access to lump sum taxation. Complete asset markets allow full insurance of consumption risk across countries. There is an implicit risk free interest rate which is common across the currency union. Firm's production and supply is constrained by Calvo style sticky prices.

2.1 Households

Utility of a representative infinitely lived home household evaluated from date 0 is:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t (U(C_t, \xi_t) - V(N_t) + J(G_t)) \tag{1}$$

where felicity is the functions U, V, and J represent the utility of the composite South consumption bundle $U(C_t, .)$, disutility of labor, $V(N_t)$, and utility of the government good $J(G_t)$, respectively with $\beta < 1$. The variable ξ_t represents a demand shock to preferences, (assuming that $U_{12} > 0$). Define $\sigma \equiv -\frac{U_{CC}\overline{C}}{U_C}$ as the inverse of the elasticity of intertemporal substitution in consumption, $\phi \equiv -\frac{V''\overline{H}}{V'}$ as the elasticity of the marginal disutility of hours worked and $\sigma_g \equiv -\frac{J''\overline{G}}{J'}$ as the elasticity of marginal utility of public goods. In addition, we assume that $\sigma_g = \sigma > 1$, consistent with empirical evidence (see, e.g. Yogo, 2004).

The composite consumption consists of a geometric average of home and foreign goods.

$$C_t = \Phi C_{St}^{v/2} C_{Nt}^{1-v/2}, \quad v \geq 1$$

where $\Phi = \left(\frac{v}{2}\right)^{\frac{v}{2}} \left(1 - \left(\frac{v}{2}\right)\right)^{\frac{v}{2}}$, C_S is South consumption of the South produced composite good , and C_N is South consumption of the North produced composite good[2]. The parameter $v \geq 1$ governs the degree of home bias in the consumption basket of each country. For $v = 1$, both countries are fully open to trade and in a symmetric equilibrium exports are %50 of GDP, while with $v = 2$, there is zero trade and the union consists of two closed economies.

Consumption aggregates, C_S and C_N are composites, defined over a range of home and foreign differentiated goods, with elasticity of substitution $\theta > 1$ between goods.

$$C_S = \left[\int_0^1 C_S(i)^{\frac{1}{1-\theta}} di \right]^{\frac{1}{1-\theta}}, \qquad C_N = \left[\int_0^1 C_N(i)^{\frac{1}{1-\theta}} di \right]^{\frac{1}{1-\theta}}.$$

The demand for good i in region $j = S, N$ is

$$\frac{C_j(i)}{C_j} = \left(\frac{P_j(i)}{P_j} \right)^{-\theta}$$

where the price indices for home and foreign goods are:

$$P_S = \left[\int_0^1 P_S(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \qquad P_N = \left[\int_0^1 P_N(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

while the aggregate CPI price index for the North region is $P = P_S^{v/2} P_N^{1-v/2}$ and for the South region is $P_t^* = P_N^{v/2} P_S^{1-v/2}$. In each region, government spending has complete home bias; agents only get utility from spending on the domestic good. Government demand for each individual variety of the home good has price elasticity θ, the same as that for private spending.

The household's implicit labor supply at nominal wage W_t is:

$$U_C(C_t, \xi_t) W_t = P_t V'(N_t). \tag{2}$$

Optimal risk sharing implies

[2] Our objective is to illustrate the main points of the analytically, Therefore, we make the simplifying assumption that the trade elasticity is unity () and the relative population sizes are equal. For more general representations, see Bhattarai and Egorov, 2016, and Erceg and Linde, 2010.

$$U_C(C_t,\xi_t) = U_C(C_t^*,\xi_t^*)\frac{P_t^*}{P_t} = U_C(C_t^*,\xi_t^*)T_t^{v-1}, \tag{3}$$

where $T = \frac{P_N}{P_S}$ is the terms of trade. Nominal bonds pay interest, R_t. Then the Euler equation is:

$$\frac{1}{R_t} = E_t\beta\left[\frac{P_t}{P_{t+1}}\frac{U_C(C_{t+1},\xi_{t+1})}{U_C(C_t,\xi_t)}\right] = E_t\beta\left[\frac{P_t^*}{P_{t+1}^*}\frac{U_C(C_{t+1}^*,\xi_{t+1}^*)}{U_C(C_t^*,\xi_t^*)}\right]. \tag{4}$$

2.2 Firms

Each firm i employs labor to produce a differentiated good.

$$Y_t(i) = N_t(i),$$

Profits are $\Pi_t(i) = P_{St}(i)Y_t(i) - \frac{\theta-1}{\theta}W_tN_t(i)$ including a subsidy to labor to eliminate steady state first order inefficiencies. Price setting follows a Calvo specification with probability of price adjustment, $1-\kappa$. Reset prices are $\widetilde{P}_{St}(i)$:

$$\widetilde{P}_{St}(i) = \frac{E_t\sum_{j=0} m_{t+j}(\beta\kappa)^j\frac{W_{t+j}}{A_{t+j}}Y_{t+j}(i)}{E_t\sum_{j=0} m_{t+j}(\beta\kappa)^jY_{t+j}(i)}. \tag{5}$$

where the stochastic discount factor $m_{t+j} = \frac{P_t}{U_C(C_t,\xi_t)}\frac{U_C(C_{t+j},\xi_{t+j})}{P_{t+j}}$. The aggregate home price index follows:

$$P_{St} = [(1-\kappa)\widetilde{P}_{St}^{1-\theta} + \kappa P_{St-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{6}$$

The foreign price is defined analogously.

2.3 Market Clearing

Equilibrium in the market for good i as

$$Y_{St}(i) = \left(\frac{P_{St}(i)}{P_{St}}\right)^{-\theta}\left[\frac{v}{2}\frac{P_t}{P_{St}}C_t + (1-\frac{v}{2})\frac{P_t^*}{P_{St}}C_t^* + G_t\right],$$

where G_t represents total South government spending. Aggregate market clearing in the South good is:

$$Y_{St} = \frac{v}{2}\frac{P_t}{P_{St}}C_t + (1-\frac{v}{2})\frac{P_t^*}{P_{St}}C_t^* + G_t. \tag{7}$$

Here $Y_{St} = V_t^{-1} \int_0^1 Y_{St}(i)di$ is aggregate home country output, where we have defined $V_t = \int_0^1 \left(\frac{P_{St}(i)}{P_{Ht}}\right)^{-\theta} di$. It follows that South region employment (employment for the representative South household) is given by $N_t = \int_0^1 N(i)di = Y_{St}V_t$.

The aggregate market clearing condition for the North good is

$$Y_{Nt} = \frac{v}{2}\frac{P_t^*}{P_{Nt}^*}C_t^* + (1-\frac{v}{2})\frac{P_t}{P_{Nt}^*}C_t^* + G_t^*, \tag{8}$$

where $N_t^* = \int_0^1 N_t^*(i)di = Y_{Nt}V_t^*$, and $V_t^* = \int_0^1 \left(\frac{P_{Nt}^*(i)}{P_{Nt}^*}\right)^{-\theta} di$.

The interest rate, R_t is set by a policy rule. An equilibrium in the world economy with positive nominal interest rates may be described by the equations (3), and then (2), (4), (5) and (6) for the South and North economy, as well as (7) and (8). For given values of V_t and V_t^*, given monetary rules (to be discussed below) and given government spending policies, these equations determine an equilibrium sequence for the variables C_t,C_t^*, W_t, W_t^*, P_{St}, P_{Nt}^*, $\widetilde{P}_{St}$, $\widetilde{P}_{Nt}^*$, R_t, and N_t, N_t^*.

3 The Linear Approximation Economy

Following the approach used in much of the literature, we work with a log-linear approximation of the model in terms of inflation and output gaps (as in Engel, 2011). The linearization is taken around an efficient, flexible price allocation with zero inflation, when there is an optimal subsidy which offsets monopoly distortions, [3]. Define $\overline{x}_t$ as the percentage deviation of the flexible price equilibrium from the non-stochastic steady state. Then define $\widetilde{x}_t$ as the percentage deviation of a given variable X_t from the efficient flexible price equilibrium. For any variable x_t, we define the world average and world relative level as $x_t^W = \frac{x_t + x_t^*}{2}$ and $x_t^R = \frac{x_t - x_t^*}{2}$. Finally, define PPI inflation as π_t.

3.1 Natural Interest Rates and the Flexible Price Economy

Wicksellian, or 'natural' interest rates are defined as the equilibrium real interest rates implied by a flexible price equilibrium of the world economy, where there are no monopolistic distortions, fiscal spending is chosen optimally, and the fiscal authority can freely implement lump-sum taxes. Equivalently these may be obtained as the value of the flexible price

[3] We assume the possibility of future constraints on monetary policy as a zero-probability event, see Adam and Billi (2006, 2007), and Nakov (2008).

nominal rate less expected PPI inflation (i.e. $r_t - E_t\pi_{t+1}$).

Optimal, unconstrained fiscal policies are defined by the trade off between providing public goods (recalling the fact that public goods are all concentrated on home varieties) and the utility cost of providing the goods.

$$V'(N_t) = J'(G_t) \tag{9}$$

Define $c_y = \frac{C}{Y}$ as the steady state share of consumption in output. In linearized form, (9) is written $\overline{g}_t = -\frac{\phi}{\sigma}\overline{n}_t$[4].

Define $\varepsilon_t = \frac{U_{C\xi}}{U_C}\ln(\xi_t)$ as the measure of a positive demand shock in the South region. We solve for the natural interest rates, as in Cook and Devereux (2011a), as functions of the average level of the demand shock, ε_t^W, and regional differentials, ε_t^R.

$$\overline{r}_t = r + \left(\frac{\phi c_y}{\phi+\sigma}\left\{\varepsilon_t^W - E_t\left[\varepsilon_{t+1}^W\right]\right\} + \phi\frac{\zeta}{\Delta_D}\left\{\varepsilon_t^R - E_t\left[\varepsilon_{t+1}^R\right]\right\}\right) \tag{10}$$

$$\overline{r}_t^* = r + \left(\frac{\phi c_y}{\phi+\sigma}\left\{\varepsilon_t^W - E_t\left[\varepsilon_{t+1}^W\right]\right\} - \phi\frac{\zeta}{\Delta_D}\left\{\varepsilon_t^R - E_t\left[\varepsilon_{t+1}^R\right]\right\}\right) \tag{11}$$

where $\Delta_D \equiv \frac{\Delta}{Dc_y}$ with $\sigma > D \equiv (\sigma v(2-v)+(1-v)^2) > 1$ and $\Delta = \phi D c_y + \phi(1-c_y)+\sigma > 1$. The parameter, $0 \leq \zeta \leq 1$, measures the intensity of home bias, $\zeta \equiv \frac{(v-1)}{D}$. In the absence of home bias, $\zeta = 0$; while under full home bias $\zeta = 1$. The interest rate in a non-stochastic steady state is equal to the rate of time preference $\overline{r} = \frac{1-\beta}{\beta}$.

Since capital markets are integrated, shocks to natural interest rates move together across countries unless the two economies are completely closed to trade (i.e. $v = 2$). This implies

$$\begin{aligned}
\overline{r}_t^W &= r + \frac{\phi c_y}{\phi+\sigma}\left\{\varepsilon_t^W - E_t\left[\varepsilon_{t+1}^W\right]\right\} \\
\overline{r}_t^R &= \phi\frac{\zeta}{\Delta_D}\left\{\varepsilon_t^R - E_t\left[\varepsilon_{t+1}^R\right]\right\}
\end{aligned}$$

We can see that risk sharing (i.e. open capital markets) ensures that in the absence of home bias (i.e. $\zeta = 0$) there are no natural interest rate differentials. But in general, the trade frictions implied by home bias lead natural interest rates to diverge in response to regionally asymmetric shocks. When there is a negative demand shock concentrated in the South the South natural interest rate falls by relatively more than that of the North.

Our analytical analysis will be based on a particular persistence process for the demand

[4]Note that, approximated around the steady state, up to a first order,$n_t \approx y_t$, $n_t^* \approx y_t^*$, so the labor gap for each country will stand in for the output gap

shocks. We assume that ε_t and ε_t^*, follow a first order process with persistence, μ. In that case, natural interest rates of the South and North economy in a competitive equilibrium with optimal government spending in both countries as in (9) are defined as:

$$\overline{n}_t^R = \frac{Dc_y}{\Delta}\zeta\varepsilon_t^R = \frac{\zeta}{\Delta_D}\varepsilon_t^R$$

3.2 Dynamic Model

We model the economy in terms of two distinct sets of equations which govern aggregate inflation, output and interest rates for the currency union and, respectively regional differentials in inflation, output, and the terms of trade. These equations each follow the canonical form of the New Keynesian model and can be examined separately.

3.2.1 Aggregate Economy

Despite the presence of multiple regions with home bias, the world average of the economy is described by the New Keynesian Phillips Curve and Euler Equation.

$$\pi_t^W = k(s+\phi)\left\{\widetilde{n}_t^W - F \cdot \widetilde{cg}_t^W\right\} + \beta E_t \pi_{t+1}^W \tag{12}$$

$$sE_t(\widetilde{n}_{t+1}^W - \widetilde{n}_t^W) - sE_t(\widetilde{cg}_{t+1}^W - \widetilde{cg}_t^W) = E_t\left(r_t - \overline{r}_t^W - \pi_{t+1}^W\right) \tag{13}$$

where k captures the degree of price stickiness; in particular $k = \frac{(1-\kappa)(1-\kappa\beta)}{\kappa}$[5]. The intertemporal elasticity is adjusted for government spending $s \equiv \frac{\sigma}{c_y}$. From (12) we can thee that F is equal the zero inflation fiscal multiplier. This can be written as as $F = \frac{s}{(s+\phi)}$, and must be less than unity. Note that, unsurprisingly, these equations do not depend on the home bias parameter v, since home bias in relative spending does not affect the aggregate dynamics in this economy. We can simplify notation by defining, $\widetilde{cg}_t^W \equiv (1-c_y)\widetilde{g}_t^W$.

Monetary policy is given by the rule which allows for the zero bound to bind;

$$r_t = \text{Max}(0, \rho + \gamma\pi_t) \tag{14}$$

This set of aggregate equations has no endogenous dynamics. In equilibrium, all of the endogenous variables are functions of the exogenous variables, which in our case will constitute the aggregate demand shock $\overline{r}_t^W$ and the fiscal gap, $\widetilde{cg}_t^W$.

[5] Throughout we assume $k < \frac{(1-\beta\mu)(1-\mu)}{\mu}F$ which is necessary for stability under a zero lower bound constraint on monetary policy. This puts a lower bound on the degree of price stickiness.

3.2.2 Regional Disparities

Given the response of the aggregate economy, we can write the separate dynamics of the world 'relative' position variables in the form of the canonical New Keynesian economy:

$$\pi_t^R = k(s_D + \phi)\left\{\widetilde{n}_t^R - \digamma_D \widetilde{cg}_t^R\right\} + \beta E_t \pi_{t+1}^R \tag{15}$$

$$s_D E_t(\widetilde{n}_{t+1}^R - \widetilde{n}_t^R) - s_D E_t(\widetilde{cg}_{t+1}^R - \widetilde{cg}_t^R) = E_t\left(-\overline{r}_t^R - \pi_{t+1}^R\right) \tag{16}$$

where $\digamma_D = \frac{s_D}{(s_D+\phi)}$. We define the intertemporal elasticity adjusted for expenditure-switching as $s > s_D \equiv \frac{s}{D} > 1$. Relative demand can be more sensitive to changes in the intertemporal price than is aggregate demand. This is due to the endogenous response of the terms of trade - intuitively a fall in the real interest rate in North will precipitate a North terms of trade deterioration which crowds in spending towards the North. Define $\widetilde{cg}_t^R \equiv (1 - c_y)\widetilde{g}_t^R$.

Notice this system does not include a parameter for monetary policy. The currency union requires that a common policy interest rate applies region wide. Since there is only one nominal interest rate, the relative interest rate equations for nominal bond rates do not impose any additional constraints on the model. Taken in isolation, these equations have multiple solutions, as shown by Benigno, Benigno and Ghironi (2008). However, a unique solution is pinned down by the additional condition on the dynamics of the real exchange rate:

$$\tau_t - \tau_{t-1} = -\pi_t^R \tag{17}$$

Due to the fixed exchange rate peg, adjustments in the terms of trade can occur only through changes in nominal prices (see Benigno, 2004). But because prices are sticky, this adjustment can occur only gradually. As a result, the lagged terms of trade acts as an additional state variable in in a single currency area.

Equilibrium in goods and financial markets imply

$$\frac{\tau_t}{2} = s_D\left[\widetilde{n}_t^R - \widetilde{cg}_t^R\right] - \frac{\phi}{\Delta_D}\zeta\varepsilon_t^R \tag{18}$$

This replaces (16) as an equilibrium condition linking relative demand, represented by the effect of output gaps on the terms of trade, to relative inflation. As Equations (18) and (17)are dynamic; regional differentials in inflation and output gaps will be endogenously persistent. In a single currency area, the terms of trade change only through slow changes in domestic prices. Thus, the aggregate currency area and the relative regional positions have fundamentally different dynamics. Asymmetric shocks will have persistent distributional effects after the aggregate effects of any shocks dissipate. As we see below, the shocks which force the region into the zero bound will (in the absence of 'forward guidance' in monetary policy) have no persistent effect on the aggregate union economy after the exit from the zero bound. But the *relative* adjustment may be prolonged. An important question is whether an optimal fiscal response will speed up or slow down the relative adjustment process.

Note that in the unit intertemporal elasticity case, $\sigma = 1$, D is invariant to the degree of home bias. But home bias will still have an important effect on relative dynamics, due to its influence on the relative natural real interest rate $\overline{r}_t^R$.

4 Fiscal Multipliers

We begin by analyzing the effects of fiscal policy. We examine the marginal effect of a fiscal gap in the South economy. Suppose that $\widetilde{cg}_t = 2G$ and $\widetilde{cg}_t^* = 0$, so that $\widetilde{cg}_t^W = \widetilde{cg}_t^R = G$. Assume that government spending follows a Markov process so that $\widetilde{cg}_{t+1}^R = G$ with probability μ and 0 otherwise. [6]

4.1 Aggregate Multipliers

We can identify the marginal impact of government spending on the aggregate economy under two separate regimes, depending upon whether the zero bound constraint does or does not bind. In the first, the interest rate is set according to the Taylor rule.

$$\begin{aligned} \widetilde{n}_t^W &= M_W^{TR} \frac{(1-\beta\mu)(1-\mu) + (\gamma-\mu)ks}{(1-\beta\mu)(1-\mu)\digamma + (\gamma-\mu)ks} \digamma \widetilde{cg}_t^W + ... \\ \digamma &< M_W^{TR} = \frac{(1-\beta\mu)(1-\mu) + (\gamma-\mu)ks}{(1-\beta\mu)(1-\mu)\digamma + (\gamma-\mu)ks} \digamma < 1 \end{aligned}$$

$\frac{(1-\beta\mu)(1-\mu)}{\mu}\digamma - k$, Due to price stickiness, the multiplier under the Taylor rule is larger than the zero-inflation multiplier $\digamma$. However, the inflationary effects of aggregate government spending increases the interest rate gap which crowds out private spending. Hence, the multiplier is less than unity.

[6] Farhi and Werning (2016) provide extensive discussion of multipliers in a currency union at the zero lower bound. Here, we emphasize the role of home bias in governing the strength of cross-country multipliers.

We can also examine the multipler if the authorities are facing negtive demand shocks which push the natural rate below the zero lower bound.

$$\begin{aligned} \widetilde{n}_t^W &= M_W^{ZLB} \digamma \widetilde{cg}_t^W + ... \\ \digamma &< 1 < M_W^{ZLB} = \frac{(1-\beta\mu)(1-\mu) - \mu k}{(1-\beta\mu)(1-\mu)\digamma - \mu k} \digamma \end{aligned}$$

In the absence of an interest rate response the multiplier is larger than 1.

4.2 Regional Differentials

Fiscal policy in one country will also create differential output responses across the currency area. Since regional disparities are invariant to monetary policy, the fiscal multiplier for the relative system $\frac{d\widetilde{n}_t^R}{d\widetilde{cg}_t^R}$ is invariant to the zero lower bound. However, the endogenous dynamics of regional disparities also create a dynamic multiplier. In order to explore the fiscal response to demand shocks, we need to solve for the dynamics of relative magnitudes.

Suppose that demand shocks follow the same Markov process as government fiscal gaps with persistence parameter μ. Then $\frac{\phi}{\Delta_D}\zeta\varepsilon_t^R = \frac{\overline{r}_t^R}{1-\mu}$ so the stable solution for the terms of trade is given as:

$$\tau_t = \lambda\tau_{t-1} - V \cdot \left[\frac{\phi}{\phi + s_D} \cdot s_D \cdot \widetilde{cg}_t^R + \frac{\overline{r}_t^R}{1-\mu}\right] \tag{19}$$

where

$$0 < V \equiv \frac{\frac{k}{\digamma_D}}{\left\{\frac{k}{2\digamma_D} + (1+\beta(1-\mu)) - \beta\lambda\right\}} = \frac{2\Xi}{\left\{\frac{\Xi}{2} + \frac{1-\beta}{2} + \beta(1-\mu) + \frac{\sqrt{[\Xi+\{1+\beta\}]^2 - 4\beta}}{2}\right\}} < 2$$

We definine the parameter $\Xi = \frac{k}{2\digamma_D} = \frac{k}{2}\left(D\frac{\phi}{s} + 1\right)$. The root λ is characterized as

$$0 < \ \lambda = \frac{[\Xi + \{1+\beta\}] - \sqrt{[\Xi + \{1+\beta\}]^2 - 4\beta}}{2\beta} < 1.$$

It is straightforward to show that the root is real, positive and within the unit interval. Persistence, $\lambda(\Xi)$, is negatively related to Ξ.

$$\lambda'(\Xi) = \frac{1}{2\beta}[1 - \frac{(\Xi + \{1+\beta\})}{\sqrt{[\Xi + \{1+\beta\}]^2 - 4\beta}}] < 0$$

Intuitively, the larger is the impact of a shock on inflation, the less persistent will be changes in the terms of trade. When prices are less sticky, k will be larger, and hence Ξ will thus be larger, which means that and shocks will be less persistent. Prices and the terms of trade will adjust more quickly. In the limit, as prices become perfectly flexible, of course there is no persistence in the terms of trade at all, since then the exchange rate regime is irrelevant. Also, when ϕ is larger, labor supply is less elastic and the output gap has a bigger impact on inflation.

Most interestingly from the perspective of this paper, the parameter Ξ is negatively associated with home bias. The closer ν is to 2, the smaller will be D and Ξ and the greater the persistence in the shock. When home bias is greater, a given appreciation in the terms of trade will translate into less expenditure switching that will have less impact on inflation, prolonging the period of adjustment.

4.2.1 Dispersion Multipliers

Dispersion in fiscal policy across regions will create persistent dispersion in output. To illustrate dispersion multipliers, we simplify by assuming zero natural interest rate differentials, $\overline{r}_t^R = 0$.

Combine (17) with (19) in the case $\overline{r}_t^R = 0$ to get

$$\left(s_D\widetilde{n}_t^R - s_D\widetilde{cg}_t^R \right) = \lambda \left(s_D\widetilde{n}_{t-1}^R - s_D\widetilde{cg}_{t-1}^R \right) - \frac{V}{2} \cdot \left[\frac{\phi}{\phi + s_D} \cdot s_D \cdot \widetilde{cg}_t^R \right]$$

which implies

$$\begin{aligned} \widetilde{n}_t^R &= \lambda\widetilde{n}_{t-1}^R + \widetilde{cg}_t^R - \lambda\widetilde{cg}_{t-1}^R - \frac{V}{s_D 2} \cdot \left[\frac{\phi}{\phi + s_D} \cdot s_D \cdot \widetilde{cg}_t^R \right] \\ &= \lambda \left[\widetilde{n}_{t-1}^R - \widetilde{cg}_{t-1}^R \right] + [1 - \frac{V}{2} \cdot \frac{\phi}{\phi + s_D}] \cdot \widetilde{cg}_t^R \end{aligned}$$

This illustrates that the government spending shocks under a single currency area will generate persistent effects on output. From these expressions, we can deduce the following propositions

Proposition 1 *The impact dispersion multiplier* $M_R^{PCT} \equiv [1 - \frac{V}{2} \cdot \frac{\phi}{\phi + s_D}]$ *is between zero and one; i.e.* $0 < M_R^{PCT} < 1$.

The value of M_R^{PCT} captures the effect of a relative government spending shock on relative output. Although the absence or presence of the zero lower bound cannot affect this response, it can at most attain a value of unity. This depends on the degree of price stickiness, captured by k. For a high value of price stickiness, relative inflation is only slightly affected by the relative spending shock, and therefore there is little terms of trade appreciation, which would deflect demand away from the country receiving the relative spending shock, so the multiplier approaches unity. On the other hand, as $k \to \infty$, the dispersion multiplier approaches the flexible price value of $\digamma_D$, which must be less than unity.

Proposition 2 *When $\sigma = 1$, the impact dispersion multiplier is invariant to the degree of home bias. When $\sigma > 1$, the impact multiplier , M_R^{PCT}, will be larger when home bias is more intense.*

When $\sigma = 1$, relative government spending affect the terms of trade and output independently of the size of home bias, as can be seen from (15), (**??**), and (**??**).

Suppose again that government spending has persistence μ, so that $\widetilde{cg}_t^R = G$ and $\widetilde{cg}_{t+j}^R = G$ with prob μ or zero for all $j \geq 0$. Conditional on the government spending shock continuing we can write

$$\widetilde{n}_{t+j}^R = \left[1 - \left(\sum_{l=0}^{j} \lambda^l\right) \frac{V}{2} \cdot \frac{\phi}{\phi + s_D}\right] G + ...$$

We call the multiplier during the duration of the spending the expansion phase dispersion multiplier $M_R^{EXP}(j)$

Proposition 3 *The expanison phase dispersion multiplier $M_R^{EXP}(j) = \left[1 - \left(\sum_{l=0}^{j} \lambda^l\right) \frac{V}{2} \cdot \frac{\phi}{\phi+s_D}\right]$ is declining in j and between $0 < M_R^{EXP}(j) < M_R^{EXP}(j-1) < 1$.*

The impact of government spending differentials on output dissipates over time. The region with a more concentrated level of government spending will experience a slow terms of trade appreciation. This will shift expenditure to the other region. However, the impact on local output always exceeds that on foreign output.

The post expansionary multiplier depends on how long the expansion lasts. Consider if the expansion ends in period $t + n + 1$. Then, we can write output as:

$$\begin{aligned}
\widetilde{n}_{t+n+1}^R &= \lambda \left[\widetilde{n}_{t+m}^R - \widetilde{cg}_{t+n}^R\right] = \lambda[M_R^{EXP}(n) - 1]G < 0 \\
\widetilde{n}_{t+n+k}^R &= \lambda^k [M_R^{EXP}(n) - 1]G < 0
\end{aligned}$$

So, the post expansionary multiplier on output differentials is negative. After the expansion, the expanding economy will have an overvalued terms of trade. The direct effect of the spending shock is eliminated, and the only remnant of the shock is a higher relative price level, reducing world demand for the country's output.

Although the dispersion multiplier is independent of monetary policy, this is not true of the multipliers for the absolute value of the response of output in each country. If the economy is at the zero lower bound when the expansion occurs, fiscal expansion in one country will increase output in the other country. Define the cross-region multiplier as the response of the North output to a fiscal expansion in South, i.e., $\frac{d\tilde{n}}{dG} = M_W - M_R$.

Proposition 4 *The cross region multiplier is always positive at the zero lower bound,* $M_W^{ZLB}(j) - M_R(j) > 0$.

A corollary of (2) and (4) is that the cross-region impact multiplier is decreasing in the level of home bias - clearly with extreme home bias the cross-region multiplier must be zero.

If the expansion occurs outside the zero lower bound, the cross-region multiplier in a currency union is more complicated. A fiscal expansion concentrated in one region will lead to a real appreciation, increasing demand for the other regions goods. However, the fiscal expansion will increase aggregate inflation which will impact aggregate interest rates for all regions of the currency area. The net effect of these countervailing effects on cross-regional demand is ambiguous. The previous proposition shows that if monetary policy is sufficiently passive (i.e. zero interest response) then the positive spillovers will dominate. However, if aggregate monetary policy is sufficiently active and home bias weakens the expenditure switching effects of exchange rate appreciation, then the negative spillovers will dominate.

Proposition 5 *If the monetary policy rule implements a zero inflation equilibrium, there exists an* ν *such that the cross-region multiplier during the expansion is negative.*

Clearly, in the periods after the expansion, the cross region multiplier will be positive as $M_R(n+k)$ will be negative for any $k \geq 0$.

4.2.2 Relative Demand Shocks

Regional differences in the natural interest rate generate inflation and output dispersion by concentrating demand within a particular region. Here, we abstract from fiscal policies, assuming for now that government spending gaps are zero. In that case, relative output follows the process

$$
\begin{aligned}
\widetilde{n}_t^R &= \lambda \left[\widetilde{n}_{t-1}^R - \frac{\overline{r}_{t-1}^R}{s_D\,(1-\mu)} \right] + \left[1 - \frac{V}{2} \right] \cdot \frac{\overline{r}_t^R}{s_D\,(1-\mu)} \\
0 &< 1 - \frac{V}{2} = \frac{1 - \beta\lambda + \beta(1-\mu)}{\left\{ \frac{1}{2}\Xi + 1 - \beta\lambda + \beta(1-\mu) \right\}} < 1
\end{aligned}
$$

A natural rate shock following a Markov process that dissipates (ex post) at period $t+n+1$ has an impact

$$
\widetilde{n}_{t+j}^R = \left[1 - \left(\sum_{l=0}^{j} \lambda^l \right) \frac{V}{2} \right] \cdot \frac{\overline{r}_t^R}{s_D\,(1-\mu)}
$$

where the inequality follows from the same logic as Proposition 3. In period $t+n+1$

$$
\begin{aligned}
\widetilde{n}_{t+n+1}^R &= \lambda \left[\widetilde{n}_{t+n}^R - \frac{\overline{r}_{t+n}^R}{s_D\,(1-\mu)} \right] = \lambda [- \left(\sum_{l=0}^{n} \lambda^l \right) \frac{V}{2}] \frac{\overline{r}_t^R}{s_D\,(1-\mu)} < 0 \\
\widetilde{n}_{t+n+k}^R &= \lambda^k [- \left(\sum_{l=0}^{n} \lambda^l \right) \frac{V}{2}] \frac{\overline{r}_t^R}{s_D\,(1-\mu)} \\
-\lambda^k [\left(\sum_{l=0}^{n} \lambda^l \right) \frac{V}{2}] &< 0
\end{aligned}
$$

So in the same manner as the response to a fiscal shock a positive demand shock will have positive but diminishing effects on relative output until the shock ends, then relative output effects go in reverse, as the affected country finds itself with an appreciated real exchange rate.

Relative inflation follows

$$
\pi_t^R = \lambda \pi_{t-1}^R + V \cdot \left[\frac{\overline{r}_t^R - \overline{r}_{t-1}^R}{(1-\mu)} \right]
$$

Again, a demand shock increases inflation differentials during the period of the shock, then reverses after the shock dissipates.

Countercyclical Policy Now, consider the impact of fiscal policy rules. In the next section, we derive the optimal welfare-maximizing fiscal gaps. Here, we assume simply that the the government implements endogenous spending differentials as a function of the output gaps, so that, for $\Phi > 0$,

$$
\widetilde{cg}_t^R = -\Phi \cdot \widetilde{n}_t^R
$$

In the case of endogenous fiscal policy, we can redefine the parameter $\overleftrightarrow{\Xi} = \Xi \cdot \frac{1+F_D\Phi}{1+\Phi} < \Xi$. The stable solution for the terms of trade has the same form as (19):

$$\tau_t = \overleftrightarrow{\lambda}\tau_{t-1} - \overleftrightarrow{V} \cdot \frac{\overline{r}_t^R}{(1-\mu)} \tag{20}$$

where the initial impact of demand shocks on the terms of trade and its persistence are a function of $\overleftrightarrow{\Xi}$

$$\begin{aligned} 0 \quad < \quad \overleftrightarrow{\lambda}\left(\overleftrightarrow{\Xi}\right) &= \frac{\left[\overleftrightarrow{\Xi} + \{1+\beta\}\right] - \sqrt{\left[\overleftrightarrow{\Xi} + \{1+\beta\}\right]^2 - 4\beta}}{2\beta} < 1 \\ \overleftrightarrow{V}\left(\overleftrightarrow{\Xi}\right) &= \frac{2\overleftrightarrow{\Xi}}{\left\{\frac{1}{2}\overleftrightarrow{\Xi} + \frac{1-\beta}{2} + \beta(1-\mu) + \frac{\sqrt{\left[\overleftrightarrow{\Xi}+\{1+\beta\}\right]^2 - 4\beta}}{2}\right\}} < 2 \end{aligned}$$

where as shown previously $\overleftrightarrow{\lambda}'(\overleftrightarrow{\Xi}) < 0$ and as implied by the proof to proposition (2). Note $\overleftrightarrow{\Xi}$ is decreasing in Φ. So, the more counter-cyclical is fiscal policy (i.e. the larger is Φ) the smaller will be the initial impact of demand shocks on the terms of trade as, as the reallocation of public sector demand across regions will offset the initial shock to private sector demand. However, at the same time demand shocks will have less impact on inflation differentials, so inflation will adjust more slowly. Hence the countercyclical public sector spending rule will slow down the adjustment to the initial demand shock.

We can write the dynamics of output as

$$\begin{aligned} \left(s_D\widetilde{n}_t^R - \frac{\overline{r}_t^R}{(1-\mu)} - s_D\widetilde{cg}_t^R\right) &= \overleftrightarrow{\lambda}\left(s_D\widetilde{n}_{t-1}^R - \frac{\overline{r}_{t-1}^R}{(1-\mu)} - s_D\widetilde{cg}_{t-1}^R\right) - \frac{\overleftrightarrow{V}}{2} \cdot \frac{\overline{r}_t^R}{(1-\mu)} \\ \left(s_D\left(1+\Phi\right)\widetilde{n}_t^R\right) &= \overleftrightarrow{\lambda}\left(s_D\left(1+\Phi\right)\widetilde{n}_{t-1}^R - \frac{\overline{r}_{t-1}^R}{(1-\mu)}\right) + \left[1 - \frac{\overleftrightarrow{V}}{2}\right] \cdot \frac{\overline{r}_t^R}{(1-\mu)} \\ \widetilde{n}_t^R &= \overleftrightarrow{\lambda}\left[\widetilde{n}_{t-1}^R - \frac{\overline{r}_{t-1}^R}{s_D\left(1+\Phi\right)(1-\mu)}\right] + \frac{\left[1 - \frac{\overleftrightarrow{V}}{2}\right]}{(1+\Phi)} \cdot \frac{\overline{r}_t^R}{s_D\left(1-\mu\right)} \end{aligned}$$

Proposition 6 *Counter-cyclical fiscal policy differentials will reduce the immediate impact of a relative demand shock on the dispersion of the output gap, but at the same time prolong the period of adjustment of the real exchange rate to the shock.*

Inflation follows

$$\pi_t^R = \overleftrightarrow{\lambda} \pi_t^R + \overleftrightarrow{V} \left[\frac{\overline{r}_t^R - \overline{r}_{t-1}^R}{(1-\mu)} \right]$$

Again, a natural interest rate differentials shock increases inflation differentials during the period of the shock, then reverses after the shock dissipates.

5 Optimal Fiscal Policy

We now turn to the analysis of optimal fiscal policy in the monetary union. As shown in Cook and Devereux (2011a), a second order approximation to an equally weighted world social welfare can also be constructed in world averages and world differences. Welfare for any period t is written as:

$$\begin{aligned} LOSS_t &= -(\widetilde{n}_t^R)^2 \cdot \frac{A}{2} - (\widetilde{n}_t^W)^2 \frac{B}{2} - (\widetilde{cg}_t^R)^2 \cdot \frac{F}{2} - (\widetilde{cg}_t^W)^2 \cdot \frac{H}{2} - J(\widetilde{n}_t^R)(\widetilde{cg}_t^R) \\ &\quad -L(\widetilde{n}_t^W)(\widetilde{cg}_t^W) - \frac{\theta}{2k}(\pi_t^W)^2 - \frac{\theta}{2k}(\pi_t^R)^2 \end{aligned} \tag{21}$$

where A, B, F, H, J, L are defined in the Appendix. Thus, the social welfare function faced by the policy maker depends upon output gaps, inflation rates, fiscal gaps, and the interaction between these variables. As in the positive analysis of the response to demand shocks described above, we can separate the optimal fiscal policy problem into a choice of world average and world relative policy instruments.

We focus on an optimal policy response without commitment. Hence, the Given this welfare function, cooperative optimal policy maximizes the objective function

$$\begin{aligned} & E_t \left[\sum_{j=0}^{\infty} \beta^j V_{t+j} \right] \\ V_t &= LOSS_t + \omega_t^W \left[\pi_t^W - k(\phi + s)\widetilde{n}_t^W + ks \cdot \widetilde{cg}_t^W - \beta E_t \pi_{t+1}^W \right] \\ &\quad + \omega_t^R \left[\pi_t^R - k(\phi + s_D)\widetilde{n}_t^R + ks_D \widetilde{cg}_t^R - \beta E_t \pi_{t+1}^R \right] \\ &\quad + \psi_t^W \left[sE_t(\widetilde{n}_{t+1}^W - \widetilde{n}_t^W) - sE_t(\widetilde{cg}_{t+1}^W - \widetilde{cg}_t^W) - E_t \left(r_t - \widetilde{r}_t^W - \pi_{t+1}^W \right) \right] \\ &\quad + \psi_t^R \left[\pi_t^R + 2\left(s_D \left[\widetilde{n}_t^R - \widetilde{cg}_t^R \right] - \frac{\phi}{\Delta_D} \zeta \varepsilon_t^R \right) - 2\left(s_D \left[\widetilde{n}_{t-1}^R - \widetilde{cg}_{t-1}^R \right] - \frac{\phi}{\Delta_D} \zeta \varepsilon_{t-1}^R \right) \right] \end{aligned}$$

where the constraints are the equation describing the dynamics of the economy and

$$r_t = \mathrm{Max}(0, \rho + \gamma\pi_t) \tag{22}$$

This implies that optimal fiscal policy

5.1 First Order Conditions

5.1.1 Aggregate Economy

The first order conditions describing optimal policy for the aggregate economy are

$$-B\widetilde{n}_t^W - L\widetilde{cg}_t^W = k(\phi + s)\omega_t^W + s\psi_t^W \tag{23}$$

$$H\widetilde{cg}_t^W + L\widetilde{n}_t^W = ks\omega_t^W + s\psi_t^W \tag{24}$$

$$\theta\pi_t^W = k\omega_t^W \tag{25}$$

$$\psi_t^W = \gamma_t$$

The constraint on r^W at the zero lower bound implies that either the shadow value, γ_t, is zero or the zero lower bound binds, r^W.

When γ_t is zero and policy rates are unconstrained by the zero lower bound, $r_t = \widetilde{r}_t^W$, and price stability is the cooperative optimal policy under both commitment and discretion, (see Benigno and Benigno, 2003); the aggregate fiscal gap will be zero along with output gap and inflation. Intuitively, if policy is determined relative to an initial steady state without monopoly distortions, and there are no mark-up shocks, optimal cooperative policy will close all gaps, whether under discretion or commitment. A policy of price stability can be implemented by setting nominal interest rates equal to natural interest rates as defined in (10) and (11). Note that in order to implement this policy, it is necessary that there be an interest rate feedback rule on inflation or other endogenous variables, in order to avoid indeterminacy (see e.g. Gali, 2008, and Benigno and Benigno, 2008).

When the zero bound on interest policy binds, there is a role for aggregate discretionary government spending in response to natural interest rate shocks. When natural interest rate shocks follow a Markov process in which the current natural interest rate is below the zero lower bound and has a probability of $(1 - \mu)$of returning permanently to a positive range, we can solve for the optimal fiscal policy in closed form.

$$cg_t^W = \frac{-\left[(1-\beta\mu)BL + \phi\theta k(\phi+s)\right](1-\beta\mu)\cdot \tilde{r}_t^W}{\left[(1-\beta\mu)\left(H + B\cdot M_W^{ZLB}\right)L + \phi\theta k(\phi + \{M_W^{ZLB}-1\}\, s)\right]\left[(1-\beta\mu)(1-\mu)s - \mu k(\phi+s)\right]} \tag{26}$$

In a liquidity trap, the fiscal multiplier $M_W^{ZLB} > 1$, so it is clear that optimal fiscal gaps are negatively associated with demand shocks.

5.1.2 Regional Differentials

We can separately solve for the optimal relative responses of fiscal policy. The first order conditions governing regional differentials are:

$$-A\tilde{n}_t^R - J(\tilde{cg}_t^R) = \omega_t^R k(\phi + s_D) - 2s_D[\psi_{t.}^R - \beta\psi_{t+1}^R) \tag{27}$$

$$F\tilde{cg}_t^R + J(\tilde{n}_t^R) = ks_D\omega_t^R - 2s_D[\psi_{t.}^R - \beta\psi_{t+1}^R) \tag{28}$$

$$k\omega_t^R + k\psi_{t.} = \theta\pi_t^R \tag{29}$$

As noted by, Corsetti, Keester and Muller (2012) and Cook and Devereux (2015), a currency union contains a commitment to maintaining stationary price level differentials. Thus, the optimal rules are dynamic in nature, even when no *fiscal* commitment is available. The first order conditions can be solved forward:

$$E_t\left[\sum_{j=0}^{\infty}\beta^{**j}\left(s_D\theta\pi_{t+j}^R - J\tilde{n}_{t+j}^R - F\tilde{cg}_{t+j}^R\right)\right] = f\cdot E_t\left[\sum_{j=0}^{\infty}\beta^{*j}\left((\phi+s_D)\theta\pi_{t+j}^R + A(\tilde{n}_{t+j}^R) + J\tilde{cg}_{t+j}^R\right)\right]$$

$$\sum_{j=0}^{\infty}\left\{\beta^{**j}F + \beta^{*j}fJ\right\}\tilde{cg}_{t+j}^R = -\sum_{j=0}^{\infty}\left\{\beta^{**j}F + \beta^{*j}fA\right\}\tilde{n}_{t+j}^R - \theta\sum_{j=0}^{\infty}\left\{\beta^{*j}f\cdot(\phi+s_D) - \beta^{**j}s_D\right\}$$

where $f = \frac{(2F_D + kF_D)}{(2F_D+k)} < 1$, $\beta^* \equiv \frac{1}{1+\Xi}\beta$ where $\beta^{**} = \frac{1}{1+F_D\Xi}\beta$.

Proposition 7 *As $\beta \to 0$, optimal relative consumption is a negative function of inflation and output gap differentials.*

$$\tilde{cg}_{t+j}^R = -\frac{\{F+fA\}}{\{F+fJ\}}\tilde{n}_{t+j}^R - \frac{\theta}{\{F+fJ\}}\frac{2F_D}{(2F_D+k)}(\phi)\pi_{t+j}^R$$

The optimal choice of fiscal policy implies a trade-off between the various distortions. If relative demand shocks are creating distortions in the relative output gap and relative inflation, then optimal government spending gaps cannot be zero. Since autoregressive relative demand shocks shift the output gap and inflation in the same direction, the fiscal gap should move in a counter-cyclical direction.

5.2 Numerical Experiments

The above results establish that the aggregate government spending gap should respond to demand shocks only if monetary policy is constrained by the zero lower bound. However, in a currency union, there is a role for regional differentials in monetary policy to mitigate against regional differentials in demand, regardless of whether the zero lower bound binds or not. We show this in a numerical solution to the optimal fiscal policy. In the numerical solution we assume there is a preference shock to the South economy which shifts the aggregate natural interest rate to $r_t^W = -.02$ (-8% on annualized basis) for a fixed number N of periods[7]. Following this, the aggregate natural interest rate shifts to the rate of home preference, $r_{t+N+1}^W = \overline{r}$ forever. Rather than assume optimal monetary policy, we simplify by assuming a policy rule

$$r_t = \text{Max}(0, \rho + \gamma\pi_t) \tag{30}$$

We search for a solution to the set of equations

$$\begin{aligned} -\left\{\tau_t - \tau_{t-1}\right\}\pi_t^R &= k(\phi + s_D)\left\{\widetilde{n}_t^R - \digamma_D \widetilde{cg}_t^R\right\} + \beta E_t \pi_{t+1}^R \\ -\left\{\tau_t - \tau_{t-1}\right\} &= k(\phi + s_D)\left\{\widetilde{n}_t^R - \digamma_D \widetilde{cg}_t^R\right\} - \beta E_t\left[\tau_{t+1} - \tau_t\right] \end{aligned}$$

where optimal fiscal policy accords with (27), (28), and (34). We search for solutions for the dynamics of the real exchange rate.

$$\tau_t = \lambda\tau_{t-1} - V \cdot \varepsilon_t^R$$

5.2.1 Benchmark Parameterization

The benchmark parameterization of preference parameters are taken from Cook and Devereux (2011a). The subjective discount factor is $\beta = 0.99$; the inverse of the Frisch elasticity

[7]Note that we do not consider infra-marginal shifts in fiscal policy that might endogenously change the duration of the liquidity trap (see Erceg and Linde, 2014).

of labor suppy, $\phi = 1$; the inverse of the intertemporal elasticity of substitution is set at $\sigma = 2$; and the elasticity of substitution between differentiated goods, $\theta = 10$.(indicating a 10% steady state markup). The steady-state government share of output is $c_y = .2$. We assume a price stickiness parameter, $\kappa = 0.825$. We base our benchmark measure of home bias, $\nu = 1.5$, which leads to a steady state imports to GDP ratio of consisting of 25%. The length of the negative demand shocks N is set to 5. To approximate optimal monetary policy , we assume a Taylor rule coefficient of $\gamma = 30$ for strong inflation stabilization.

We examine the effects of the South demand shock under three fiscal rules: 1) no fiscal policy gap (i.e. $\widetilde{cg}_t = \widetilde{cg}_t^* = 0$); 2) optimal fiscal policy (i.e. solve 23-25 & 27-34);. and 3) a rule of thumb counter-cyclical policy (i.e. $\widetilde{cg}_t = -\Phi\ \widetilde{y}_t$, $\widetilde{cg}_t^* = -\Phi\ \widetilde{y}_t^*$);

Figure 1 illustrates the response of the aggregate economy to a demand shock in the home region. Figure 1, Panel A shows the effect on the aggregate natural interest rate. By construction, the natural interest rate shifts to negative eight hundred annualized basis points for 5 periods before returning back to steady state. Panel B show the dynamics of the nominal interest rate which go to zero for 5 periods then returns to near steady state after the shock dissipates. Panel C shows that the decline in aggregate demand. Consumption demand declines immediately upon the shock and converges smoothly back to steady-state by the time the shock dissipates (see Panel F). As shown in Panel D, the demand shock is disinflationary, pushing aggregate inflation down in the period of the shock; inflation converges back to steady state until the shock disappears. Panel E shows the response of optimal government spending. Optimal government spending works to offset the decline in demand as in (26) under slightly different exogenous dynamics. The aggregate fiscal gap increases persistently as private demand declines. We examine the economy with $\Phi = .5$; such a counter-cyclical policy in fact closely matches optimal aggregate policy and could be easily implemented. In either case, the decline in the output gap and inflation are smaller than when no fiscal policy is implemented. However, because fiscal policy is itself distortionary, the fiscal expansion is not sufficient to close the output gap or stabilize prices.

Figure 2 demonstrates the response of regional differentials to a demand shock in the home region. A spread opens up in the natural interest rate across regions. As the demand shock is concentrated on the South due to home bias, the natural rate falls more in the South than in the North. In this example, the natural interest rate declines by about 500 annualized basis points more in the South than the North. The dispersion in demand conditions leads to a corresponding dispersion in the output gap and inflation with the demand shock in the South putting stronger downward pressure on South output and inflation. The excess disinflation in the South leads to a real depreciation for the South. Over time, the adjustment of the real exchange rate helps equalize demand. Inflation and the output gap converge back to zero until the negative demand shock dissipates. In period 6, when the demand shock disappears, the strong North real exchange rate will shift relative demand in the opposite direction. Over time, the real exchange rate will converge back to steady-state. The long-term effects on, τ_t, will also continue to distort regional allocations of output and inflation following the decline in the shock.

Optimal fiscal policy is to shift the path of the relative fiscal gap against relative demand. There is an increase in the dispersion of fiscal spending with optimal spending concentrated in the South. Given home bias of government expenditure, tilting fiscal policy toward demand deficient regions helps to reduce the impact of distortions. Note this regional dispersion occurs regardless of whether the aggregate economy is at the zero lower bound. In normal times, when monetary policy is free to address aggregate demand, optimal regional fiscal policies will always address demand differentials through expansions in one region moving accompanied by contractions in opposing regions. At the zero lower bound, Northern fiscal policy will be subject to opposing influences; the standard tendency to contract in order to reallocate demand toward the more adversely affected region will balance with a need to expand demand through out the currency union. Compare the optimal fiscal policy with the counter-cyclical policy with $\Phi = .5$. In this particular example, this counter-cyclical policy delivers aggregate fiscal spending that is close to optimal. However, counter-cyclical policy tilts spending toward the South noticeably less than optimally. Simple counter-cyclical policy does not create enough regional differentials in expenditure.

Figure 2 also illustrates the results of Proposition 6 - the optimal fiscal gap response mitigates the initial real exchange rate depreciation for the South economy but also prolongs the period of adjustment to the shock.

Figure 3 shows the response of the regional economies to the South demand shock. In the South, the negative demand shock pushes the natural interest rate below -12% on an annualized basis for 5 periods. This leads to a persistent contraction in output and inflation. Optimal fiscal policy unambiguously responds to the decline in the Southern economy with an expansion. Interestingly, a pure counter-cyclical policy with $\Phi = .5$ would be over expansionary in the Southern region. In the North, the Southern demand shock pushes down demand for Northern goods (to a lesser extent) and pushes the natural rate to about -3.5 percent on an annualized basis. The demand spillovers push the North output gap and inflation into persistent negative territory. Simple national level counter-cyclical would imply a Northern fiscal expansion. However, cooperative policy goals imply a mild Northern fiscal contraction which would optimally shift demand toward the South. This would lead to a mildly sharper output gap contraction in the North. Note that the Northern fiscal contraction reverses after a few periods. This is also true of the South fiscal policy - the initial expansion is countered by a contraction following the elapsing of the shock. This reversal of the stance of policy is a consequence of the inherent persistence of terms of trade adjustment in the currency union. After the initial shock dissipates, the South region has an excessively depreciated terms of trade, and the optimal fiscal stance is to contract the South fiscal gap and to expand the North fiscal gap so as to facilitate the adjustment back to the steady state terms of trade.

In addition, we see here the important additional dimension that is brought to the regional fiscal responses by the presence of the zero lower bound. Absent the zero lower bound, optimal fiscal responses within the currency union should always be negatively correlated. The fiscal gap in the most affected area should rise, while that in the least affected area should fall. But with a region-wide liquidity trap, it is more likely than an optimal response will involve joint expansion in both regions, although it is always the case that the expansion in the most affected region is larger. Despite this, it remains the case that the absolute difference between the optimal fiscal responses in South and North depends only on the differential severity of the shock (which is determined by the degree of home bias), and is independent of the overall size of the shock. Hence, a shock which hits the South primarily should always be followed by a differentially greater expansion in South than North by the same amount, whatever the size of the overall shock.

5.2.2 Robustness Checks

To illustrate the balance between the driving motivations of North fiscal policy, regional reallocation and aggregate expansion, we consider the model at different parameterizations. Figure 4 reports the optimal North fiscal expansion under varying parameters. First, we consider different levels of home bias. The parameter ν measures home bias. The closer that ν is to one, the more integrated are the two economies.

- Panel A shows that when ν is sufficiently close to 1 (in this case, when $\nu = 1.25$), the optimal North fiscal policy is expansionary. First, when home bias is less pronounced there will be stronger demand spillovers across regions and, thus, less regional dispersion requiring dispersion in fiscal policy. Intuitively, with zero home bias, the demand shock is felt uniformly across both regions and will require an equivalent expansionary response of the fiscal gap in both regions. Second, the cross-region multiplier will be stronger when home bias in demand is relatively less, making North fiscal policy more effective in treating a demand spillover.

- Panel B examines the response of the North fiscal gap under different parameterizations of the intertemporal elasticity of substitution. In each parameterization, North fiscal policy starts out contractionary to reallocate demand toward the most affected economy but switches toward expansion as the real exchange rate slowly fills the role of demand reallocation as it adjusts through inflation differentials. The higher is intertemporal substitution, the more sensitive is demand to these real exchange rate dynamics and the more pronounced are the dynamics of appropriate North fiscal policy. When intertemporal substitution is elastic, the initial contraction in North fiscal policy is sharper but the shift into expansion comes earlier and is more marked.

- Panel C shows North fiscal policy under different specifications of the price stickiness parameter κ. When prices adjust more quickly, demand rellocation through real exchange rate adjustment will be faster and aggregate multipliers larger. Thus, the need to use fiscal policy to reallocate demand will be smaller and the incentive to use fiscal policy to address the demand short-fall will be larger. When prices are sufficiently flexible (in this case, when $\kappa = .75$), the optimal North fiscal policy will be expansionary. When prices are sufficiently sticky, the optimal fiscal policy will be more contractionary than the benchmark.

- Panel D shows optimal policy under different parameterizations of the Frisch elasticity of labor supply. When labor is elastic, the aggregate multiplier will be relatively stronger and fiscal policy is more effective. If the elasticity of labor supply is sufficiently large (in this case $\phi = 2.5$), the optimal North fiscal policy will be expansionary.

- Panel E shows that optimal North monetary policy depends very little on, θ, the elasticity of substitution between goods which governs the cost of inflation volatility, or of, γ. the strength of inflation targeting after the end of the shock.

- Panel F shows optimal North fiscal policy when the shock lasts for either 2 or 8 periods. When the shock is two periods, the effect of the shock on the economy will be relatively weak and the North fiscal response will be contractionary, though weaker than that for a shock that lasts 5 periods. When the shock is relatively long-lived, however, the optimal fiscal policy response will be expansionary. Essentially, when the demand contraction is sufficiently long-lived, firms that change their prices will make large adjustments. This leads to a relatively larger shift in the real exchange rate which helps reallocate demand, allowing fiscal policy to shift toward accommodating the aggregate demand shortfall.

5.2.3 Fiscal Capacity

Given the offsetting motivations for North fiscal policy at the zero lower bound, we find that optimal fiscal policy for North might be either expansionary or contractionary. We also consider the case when South fiscal policy is constrained (perhaps by budgetary issues). Suppose that $\widetilde{cg}_t$ was set at zero. It might be interesting to identify the optimal North fiscal policy response when one region's fiscal policy is the only fiscal instrument, keeping in mind that the demand short-fall is concentrated in the South. We solve for the constrained optimal policy by maximizing the objective function with respect to an additional constraint binds:

$$\widetilde{cg}_t^W + \widetilde{cg}_t^R = \widetilde{cg}_t = 0$$

In this particular case, the system of aggregate union wide equations are no longer separate from the first order conditions governing regional dispersion.

Figure 5 shows the response of the regional economies under a North only fiscal policy along with the responses under both optimal two sided fiscal policy and zero fiscal policy. We find a relatively strong result for optimal North fiscal policy. Figure 5, Panel (A) and Panel (E) display the differential in natural interest rates across regions. Panel (D) and Panel (H) shows the fiscal policy in the North and South region. By construction, South fiscal policy is constrained relative to the optimal expansionary policy. In the absence of expansionary South fiscal policy, the optimal policy of the North region shifts toward expansion as the only instrument available to address the negative demand shortfall. The North fiscal expansion increases regional demand. When South fiscal policy is constrained, the concentration of fiscal policy in the Northern region creates an expansion of output in the Northern region. The spillover to the Southern region is small. The response of the South output gap is not much different than under no fiscal policy in either country.

North fiscal expansion is a constrained optimal result in a wide variety of circumstances. Figure 6 shows the response of the north fiscal gap in the constrained optimal under the set of parameterizations discussed in Figure 4. In all of them, the union relies on, $\widetilde{cg}_t^{North}$ to expand union wide demand. Though, although unconstrained fiscal policy may be expansionary or contractionary, when South fiscal policy is constrained, optimal North fiscal policy should be expansionary.

6 Conclusion

When a large negative demand shock constrains policy interest rates at the zero lower bound, optimal fiscal policy will be expansionary. When home bias concentrates the demand shock within a region, optimal policy will also be concentrated in that region. If the aggregate expansion is sufficiently large and the degree of concentration is sufficiently small, then both regions should expand fiscal spending. This might typically be characterized by a large aggregate contraction with relatively small regional differentials. On the other hand, if the degree of concentration is sufficiently intense as in the case of sufficient home bias in spending, fiscal policy amongst regions should diverge. This may be the case even if trade and financial channels cause both countries to experience deflatonary contractions. The above analysis assumes that both countries have unconstrained fiscal policy. If a less exposed region of the currency union was the only region with the fiscal space to implement fiscal policy, it might need towar direct spending to the aggregate decline at the cost of exacerbating regional differences.

7 Appendix

7.1 Proofs

Proof for Proposition 1

Proof.

$$\frac{V}{2}\cdot\frac{\phi}{\phi+s_D}=\frac{\Xi}{\left\{\frac{1-\beta}{2}+\beta(1-\mu)+\frac{\Xi+\sqrt{[\Xi+\{1+\beta\}]^2-4\beta}}{2}\right\}}\cdot\frac{\phi}{\phi+s_D}$$

This is $[\Xi+\{1+\beta\}]^2-4\beta=[\Xi+\{1-\beta\}]^2+4\beta>(\Xi)^2$ so $\Xi+\sqrt{[\Xi+\{1+\beta\}]^2-4\beta}>2\times\Xi$. In absolute terms, the numerator of $\frac{V}{2}$ is larger than the denominator, $1-\digamma_D=\frac{\phi}{\phi+s_D}<1$. So $0<M_R^{PCT}=[1-\frac{V}{2}\frac{\phi}{\phi+s_D}]<1$. ■

Proof for Proposition 2

Proof. We write $\frac{V}{2}\cdot\frac{\phi}{\phi+s_D}=\frac{V}{2}\cdot[1-\digamma_D]$, so $M_R^{PCT}=1-\frac{V}{2}\cdot[1-\digamma_D]=[1-\frac{V}{2}]+\frac{V}{2}\cdot\digamma_D$. Define $g(\Xi)\equiv\sqrt{[\Xi+\{1+\beta\}]^2-4\beta}$ where $g'(\Xi)=\frac{\Xi+\{1+\beta\}}{g(\Xi)}>0$. Also define $S=1-\beta+2\beta(1-\mu)>0$. We can write $\frac{V}{2}\cdot\digamma_D=\frac{k}{\{\Xi+1-\beta+2\beta(1-\mu)+g(\Xi)\}}$ is a declining function of Ξ. We can write

$$[1-\frac{V}{2}] = h(\Xi) = 1-\frac{2\Xi}{\{\Xi+S+g(\Xi)\}}$$
$$= \frac{S+g(\Xi)-\Xi}{\{\Xi+S+g(\Xi)\}}$$

So that

$$\begin{aligned}
h'(\Xi) &= \frac{(g'(\Xi)-1)\{\Xi+S+g(\Xi)\}-(g'(\Xi)+1)\{S+g(\Xi)-\Xi\}}{\{\Xi+S+g(\Xi)\}^2} \\
&= \frac{(g'(\Xi))\left[\{\Xi+S+g(\Xi)\}-\{S+g(\Xi)-\Xi\}\right]+\left[\{\Xi-S-g(\Xi)\}-\{\Xi+S+g(\Xi)\}\right]}{\{\Xi+S+g(\Xi)\}^2} \\
&= \frac{(g'(\Xi)2\Xi)-2\{S+g(\Xi)\}}{\{\Xi+S+g(\Xi)\}^2} = 2\frac{g(\Xi)}{g(\Xi)}\frac{(g'(\Xi)-\{S+g(\Xi)\}}{\{\Xi+S+g(\Xi)\}^2} \\
&= 2\frac{(g(\Xi)g'(\Xi)\Xi-g(\Xi)\{S+g(\Xi)\}}{g(\Xi)\{\Xi+S+g(\Xi)\}^2} = 2\frac{[g(\Xi)g'(\Xi)\Xi-g(\Xi)^2]-g(\Xi)S}{g(\Xi)\{\Xi+S+g(\Xi)\}^2}
\end{aligned}$$

Note $g(\Xi)g'(\Xi)\Xi = \Xi^2+\{1+\beta\}\Xi$, and $g(\Xi)^2 = [\Xi^2+2\{1+\beta\}\Xi]+(1-\beta)^2]$ which implies

$$g(\Xi)g'(\Xi)\Xi-g(\Xi)^2 = -\{1+\beta\}\Xi-(1-\beta)^2$$

$$h'(\Xi) = 2\frac{-\{1+\beta\}\Xi-(1-\beta)^2-g(\Xi)S}{g(\Xi)\{\Xi+S+g(\Xi)\}^2} < 0$$

So M_R^{PCT} is declining in $\Xi = \frac{k}{2F_D} = \frac{k}{2}\frac{\phi+s_D}{s_D} = \frac{k}{2}\frac{\phi D+s}{s}$, and thus declining in D. Since D is declining in ν when $1<\nu\leq 2$, then M_R^{PCT} so will be larger when home bias is larger. ■

Proof for Proposition 3

Proof. We can write $M_R^{EXP}(j) = M_R^{EXP}(j-1)-\lambda^j\cdot\frac{V}{2}\cdot G < M_R^{EXP}(j-1)$. For any j, $M_R^{EXP}(j) > M_R^{EXP}(\infty) = \left[1-\frac{1}{1-\lambda}\frac{V}{2}\cdot\frac{\phi}{\phi+s_D}\right]$. We can write

$$\begin{aligned}
1-\lambda &= \frac{2\beta}{2\beta}-\frac{[\Xi+\{1+\beta\}]-\sqrt{[\Xi+\{1+\beta\}]^2-4\beta}}{2\beta} \\
&= \left(\sqrt{[\Xi+\{1-\beta\}]^2+4\beta\Xi}-[\Xi+\{1-\beta\}]\right)\frac{1}{2\beta}
\end{aligned}$$

and

$$\frac{V}{2} = \frac{\Xi}{\left\{\frac{\Xi}{2}+\frac{1-\beta}{2}+\beta(1-\mu)+\frac{\sqrt{[\Xi+\{1+\beta\}]^2-4\beta}}{2}\right\}}$$
$$= \frac{2\Xi}{\left\{[\Xi+\{1-\beta\}]+2\beta(1-\mu)+\sqrt{[\Xi+\{1-\beta\}]^2+4\beta\Xi}\right\}}$$

Define

$$Y=[\Xi+\{1-\beta\}] \quad Z=\sqrt{Y^2+4\beta\Xi} \quad R=2\beta(1-\mu)$$

$$\frac{1}{1-\lambda} = \frac{2\beta}{Z-Y} \quad \frac{V}{2} = \frac{2\Xi}{Z+Y+R}$$
$$\frac{1}{1-\lambda}\frac{V}{2} = \frac{4\beta\Xi}{(Z-Y)(Z+Y+R)} = \frac{4\beta\Xi}{(Z-Y)(Z+Y)+R(Z-Y)}$$
$$= \frac{4\beta\Xi}{(Z^2-Y^2)+R(Z-Y)} = \frac{4\beta\Xi}{4\beta\Xi+R(Z-Y)} = \frac{4\Xi}{4\Xi+\frac{2\beta(1-\mu)}{\beta}(Z-Y)}$$
$$= \frac{4\Xi}{4\Xi+4(1-\mu)(1-\lambda)} = \frac{\Xi}{\Xi+(1-\mu)(1-\lambda)} < 1$$

So $\frac{1}{1-\lambda}\frac{V}{2}[1-F_D] < 1$ ■

Proof for Proposition 4

Proof. The zero bound multiplier M_W^{ZLB} exceeds one for $j \leq n$ and is zero for $j > n$. The differentials multiplier $M_R(j) < 1$ for $j \leq n$ and is zero for $j > n$. So $M_W^{ZLB}(j) > M_R(j)$ for all j. ■

Proof for Proposition 5

Proof. If monetary policy implements zero inflaiton in every period (i.e. $\gamma \to \infty$), the aggregage multiplier $M_W = F$. The long run multiplier is $M_R(\infty) = (1 - \frac{1}{1-\lambda}\frac{V}{2}\cdot\frac{\phi}{\phi+s_D}) = \left[1-\frac{1}{1-\lambda}\frac{V}{2}\cdot(1-F_D)\right] = \frac{1}{1-\lambda}\frac{V}{2}\cdot F_D + (1-\frac{1}{1-\lambda}\frac{V}{2})$. Calculate$1-\frac{V}{2}$

$$\frac{1}{1-\lambda}\frac{V}{2} = \frac{\Xi}{\Xi+(1-\mu)(1-\lambda)}$$
$$1-\frac{1}{1-\lambda}\frac{V}{2} = \frac{(1-\mu)(1-\lambda)}{\Xi+(1-\mu)(1-\lambda)}$$

and $M_R(\infty) = \frac{1}{1-\lambda}\frac{V}{2}F_D + (1-\frac{1}{1-\lambda}\frac{V}{2})$

$$M_R(\infty) = \frac{1}{1-\lambda}\frac{V}{2}F_D + (1-\frac{1}{1-\lambda}\frac{V}{2}) = \frac{\Xi F_D+(1-\mu)(1-\lambda)}{\Xi+(1-\mu)(1-\lambda)}$$

Rewrite $M_W = \digamma$

$$M_W = \digamma = \frac{\Xi \digamma + \digamma(1-\mu)(1-\lambda)}{\Xi + (1-\mu)(1-\lambda)}$$

Calculate

$$M_W - M_R(\infty) = \frac{\Xi(\digamma - \digamma_D) + (\digamma - 1)(1-\mu)(1-\lambda)}{\Xi + (1-\mu)(1-\lambda)}$$

At $\nu = 2$, $D = 1$ and $\digamma = \digamma_D$. Since $\sqrt{[\Xi + \{1+\beta\}]^2 - 4\beta} > \Xi$ and $(\digamma - 1) < 0$, if $\nu = 2$ then $M_W - M_R(\infty) < 0$. By continuity, there exists $\nu < 2$ where $M_W - M_R(\infty) < 0$. Since$M_R(\infty) < M_R(j)$ for any finite j then $M_W - M_R(j) < 0$. ■

Proof for Proposition 6

Proof. The impact effect is a function of Φ

$$\frac{h\left(\overleftrightarrow{\Xi}\right)}{(1+\Phi)} = \frac{\left[1 - \frac{\overleftrightarrow{V}}{2}\right]}{(1+\Phi)}$$

where $h(.)$ is defined as in the proof of Proposition *2*, $h'(.) < 0$. The impact multiplier is a function of Φ whose derivative is written as

$$\frac{h'\left(\overleftrightarrow{\Xi}\right)\frac{\digamma_D - 1}{(1+\Phi)^2}\Xi(1+\Phi) - h\left(\overleftrightarrow{\Xi}\right)}{(1+\Phi)^2} = \frac{-h'\left(\overleftrightarrow{\Xi}\right)\overleftrightarrow{\Xi}\frac{1-\digamma_D}{(1+\digamma_D\Phi)} - h\left(\overleftrightarrow{\Xi}\right)}{(1+\Phi)^2} < \frac{-h'\left(\overleftrightarrow{\Xi}\right)\overleftrightarrow{\Xi} - h\left(\overleftrightarrow{\Xi}\right)}{(1+\Phi)^2}$$

where the last inequality follows since $-h'\left(\overleftrightarrow{\Xi}\right)\overleftrightarrow{\Xi} > -h'\left(\overleftrightarrow{\Xi}\right)\overleftrightarrow{\Xi}\frac{1-\digamma_D}{(1+\digamma_D\Phi)} > 0$. From the proof of Proposition *2*

$$[1 - \frac{V}{2}] = h(\Xi) = \frac{S + g(\Xi) - \Xi}{\{\Xi + S + g(\Xi)\}} = \frac{g(\Xi)^2 - \Xi^2 + S^2 + 2g(\Xi)S}{\{\Xi + S + g(\Xi)\}^2}$$

where $S \equiv 1 - \beta + 2\beta(1-\mu)$ and $g(\Xi) \equiv \sqrt{[\Xi + \{1-\beta\}]^2 + 4\beta\Xi}$ so

$$g(\Xi)^2 = [\Xi + \{1-\beta\}]^2 + 4\beta\Xi = \Xi^2 + [2\{1-\beta\} + 4\beta]\,\Xi + \{1-\beta\}^2$$

$$\begin{aligned} g(\Xi)^2 - \Xi^2 + S^2 + 2g(\Xi)S \;&=\; [2\{1-\beta\} + 4\beta]\,\Xi + \left[\{1-\beta\}^2 + S^2\right] + 2g(\Xi)S = \\ &\quad [2S + 4\beta\mu]\,\Xi + \left[\{1-\beta\}^2 + S^2\right] + 2g(\Xi)S \end{aligned}$$

From the proof of Proposition*2*, $h'(\Xi) = 2\frac{\left[g'(\Xi)\Xi - g(\Xi)^2\right] - S}{\{\Xi + S + g(\Xi)\}^2}$

$$-h'(\Xi)\Xi = \frac{2S\Xi + 2g(\Xi)\Xi - 2g'(\Xi)\Xi^2}{\{\Xi + S + g(\Xi)\}^2}$$

So we can write

$$-h'(\Xi)\Xi - h(\Xi) = \frac{2S\Xi + 2g(\Xi)\Xi - 2g'(\Xi)\Xi^2}{\{\Xi + S + g(\Xi)\}^2} - \frac{[2S + 4\beta\mu]\,\Xi + [\{1-\beta\}^2 + S^2] + 2g(\Xi)S^2}{\{\Xi + S + g(\Xi)\}^2}$$

Calculating

$$\begin{aligned} &= \frac{2g(\Xi)(\Xi - S) - 2g'(\Xi)\Xi^2 - 4\beta\mu\Xi - [\{1-\beta\}^2 + S^2]}{\{\Xi + S + g(\Xi)\}^2} \\ &> \frac{2g(\Xi)(\Xi - S) - 2g'(\Xi)\Xi^2 - 4\beta\mu\Xi}{\{\Xi + S + g(\Xi)\}^2} \\ &= \frac{2g(\Xi)^2(\Xi - S) - 2g(\Xi)g'(\Xi)\Xi^2 - 4\beta\mu\Xi g(\Xi)}{g(\Xi)\,\{\Xi + S + g(\Xi)\}^2} \end{aligned}$$

Note $g(\Xi)g'(\Xi)\Xi = \Xi^2 + \{1+\beta\}\Xi$, and $g(\Xi)^2 = [\Xi^2 + 2\{1+\beta\}\Xi] + (1-\beta)^2]$ which implies

$$\begin{aligned} & g(\Xi)^2(\Xi - S) - g(\Xi)g'(\Xi)\Xi^2 \\ = & \left[\Xi^3 + 2\{1+\beta\}\Xi^2\right] + (1-\beta)^2\Xi] - \Xi^3 - \{1+\beta\}\Xi^2 \\ & -S\left[\Xi^2 + 2\{1+\beta\}\Xi\right] + (1-\beta)^2] \\ = & \{1+\beta\}\Xi^2 + (1-\beta)^2\Xi - S\left[\Xi^2 + 2\{1+\beta\}\Xi\right] + (1-\beta)^2] \\ = & [\{1+\beta\} - S]\Xi^2 + \left[(1-\beta)^2 - 2S\{1+\beta\}\right]\Xi - S(1-\beta)^2 \end{aligned}$$

We know $[\{1+\beta\} - S] = [\{1+\beta\} - 1 - \beta - 2\beta(1-\mu)] = 2\beta\mu$ and $(1-\beta)^2 - 2S\{1+\beta\} = (1-\beta)[(1-\beta) - 2(1+\beta) - 2\beta(1-\mu)] = -(1-\beta)[(1+3\beta+2\beta(1-\mu)] < 0$, so

$$\begin{aligned} & \frac{2g(\Xi)^2(\Xi - S) - 2g(\Xi)g'(\Xi)\Xi^2 - 4\beta\mu\Xi g(\Xi)}{g(\Xi)\,\{\Xi + S + g(\Xi)\}^2} \\ = & \frac{4\beta\mu\,[\Xi^2 - \Xi g(\Xi)] - (1-\beta)[(1+3\beta+2\beta(1-\mu)]\Xi - S(1-\beta)^2}{g(\Xi)\,\{\Xi + S + g(\Xi)\}^2} \\ > & -h'(\Xi)\Xi - h(\Xi) \end{aligned}$$

Since$g(\Xi) > \Xi$, then$[\Xi^2 - \Xi g(\Xi)] < 0$ so all of the elements are less than zero. Thus, the impact effect of demand shocks is a declining function of Φ when$\Phi > 0$. ■

Proof for Proposition 7

Proof. The limit can be written as:

$$\{F + fJ\}\,\widetilde{cg}_t^R = -\{F + fA\}\,\widetilde{n}_t^R - \theta\,\{f\cdot\phi - (1-f)s_D\}\,\pi_t^R$$

Write the parameter ■

Proof.

$$1 - f = (1 - F_D)\frac{k}{(2F_D + k)} = \frac{(\phi)}{(\phi + s_D)}\frac{k}{(2F_D + k)}$$

Solving for the coefficient on inflation:

$$\begin{aligned}
(1-f)s_D &= \frac{(\phi)}{(\phi + s_D)}\frac{k}{(2F_D + k)}s_D = \frac{s_D}{(\phi + s_D)}\frac{k}{(2F_D + k)}(\phi) \\
&= \frac{F_D k}{(2F_D + k)}(\phi) \\
\{f\cdot\phi - (1-f)s_D\} &= \frac{(2F_D + kF_D)}{(2F_D + k)}(\phi) - \frac{F_D k}{(2F_D + k)}(\phi) = \frac{2F_D}{(2F_D + k)}(\phi)
\end{aligned}$$

■

7.2 Parameter Derivations

Here we define the parameters used in the loss function, which is taken from that used in Cook and Devereux (2011).

$$\begin{aligned}
A &\equiv \left\{\frac{(1+\phi c_y)}{c_y^2} + \frac{(\sigma - D)}{D}(1 + \frac{(1-c_y^2)}{c_y^2 D})\right\} = \frac{(s_{DD} + \phi)}{c_y} \\
s_{DD} &\equiv \frac{(D-1)(1-c_y^2)}{c_y D} + \frac{(\sigma)}{D}(\frac{1 + c_y^2(D-1)}{c_y D}) < s_D \\
B &\equiv \frac{(\sigma + \phi c_y)}{c_y^2} = \frac{(s+\phi)}{c_y}, \\
H &\equiv \frac{1}{(1-c_y)}\frac{\sigma}{c_y^2} = \frac{1}{(1-c_y)}\frac{s}{c_y} \quad L \equiv \frac{-\sigma}{c_y^2} = -\frac{s}{c_y} \\
J &\equiv \left[-\frac{1}{c_y^2} - \frac{(\sigma - D)}{c_y^2 D^2}(1 + (v-1)(D-1)c_y^2)\right] \\
F &\equiv \frac{((1-c_y) + c_y\sigma)}{(1-c_y)c_y^2} + \frac{(\sigma - D)}{c_y^2 D^2}(1 + (v-1)(D-1)c_y^2)
\end{aligned}$$

Given this, cooperative optimal policy maximizes the objective function

$$E_t\left[\sum_{j=0}^{\infty}\beta^j V_{t+j}\right]$$

$$\begin{aligned}
V_t &= LOSS_t+\omega_t^W\left[\pi_t^W-k(\phi+s)\widetilde{n}_t^W+ks\cdot\widetilde{cg}_t^W-\beta E_t\pi_{t+1}^W\right] \\
&+\omega_t^R\left[\pi_t^R-k(\phi+s_D)\widetilde{n}_t^R+ks_D\widetilde{cg}_t^R-\beta E_t\pi_{t+1}^R\right] \\
&+\psi_t^W\left[sE_t(\widetilde{n}_{t+1}^W-\widetilde{n}_t^W)-sE_t(\widetilde{cg}_{t+1}^W-\widetilde{cg}_t^W)-E_t\left(r_t-\widetilde{r}_t^W-\pi_{t+1}^W\right)\right] \\
&+\psi_t^R\left[\pi_t^R+2\left(\ s_D\left[\widetilde{n}_t^R-\widetilde{cg}_t^R\right]-\frac{\overline{r}_t^R}{(1-\mu)}\right)-2\left(\ s_D\left[\widetilde{n}_{t-1}^R-\widetilde{cg}_{t-1}^R\right]-\frac{\overline{r}_{t-1}^R}{(1-\mu)}\right)\right] \\
&+\gamma_t r_t
\end{aligned}$$

$$\begin{aligned}
LOSS_t &= -(\widetilde{n}_t^R)^2\cdot\frac{A}{2}-(\widetilde{n}_t^W)^2\frac{B}{2}-(\widetilde{cg}_t)^2\cdot\frac{F+H}{2}+J(\widetilde{n}_t^R)(\widetilde{cg}_t) \\
&-L(\widetilde{n}_t^W)(\widetilde{cg}_t)-\frac{\theta}{2k}(\pi_t^W)^2-\frac{\theta}{2k}(\pi_t^R)^2
\end{aligned} \tag{31}$$

$$\{F+H\}\,\widetilde{cg}_t-J(\widetilde{n}_t^R)+L\widetilde{n}_t^W=ks\omega_t^W+s\psi_t^W+ks_D\omega_t^R-2s_D[\psi_{t.}^R-\beta\psi_{t+1}^R) \tag{32}$$

$$-B\widetilde{n}_t^W-L\widetilde{cg}_t=k(\phi+s)\omega_t^W+s\psi_t^W$$

$$\theta\pi_t^W=k\omega_t^W$$

$$\psi_t^W=\gamma_t$$

$$-A\widetilde{n}_t^R+J(\widetilde{cg}_t^R)=\omega_t^R k(\phi+s_D)-2s_D[\psi_{t.}^R-\beta\psi_{t+1}^R) \tag{33}$$

$$k\omega_t^R+k\psi_{t.}=\theta\pi_t^R \tag{34}$$

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Figure 1 shows the response of the aggregate economy to a shock to demand shock concentrated in the South region that lasts for 5 periods. The shock is constructed to bring the aggregate natural interest rate to -8 percent on an annualized rate. The figure shows the response when: *Optimal* both regions implement optimal cooperative fiscal policy; *Countercyclical* each region sets the fiscal policy gap as a constant negative ratio of the output gap; and *Zero Gap* when the fiscal gap in each region is set to zero. Panel (B) shows the nominal interest rate which goes to the zero lower bound during the periods of the shock.

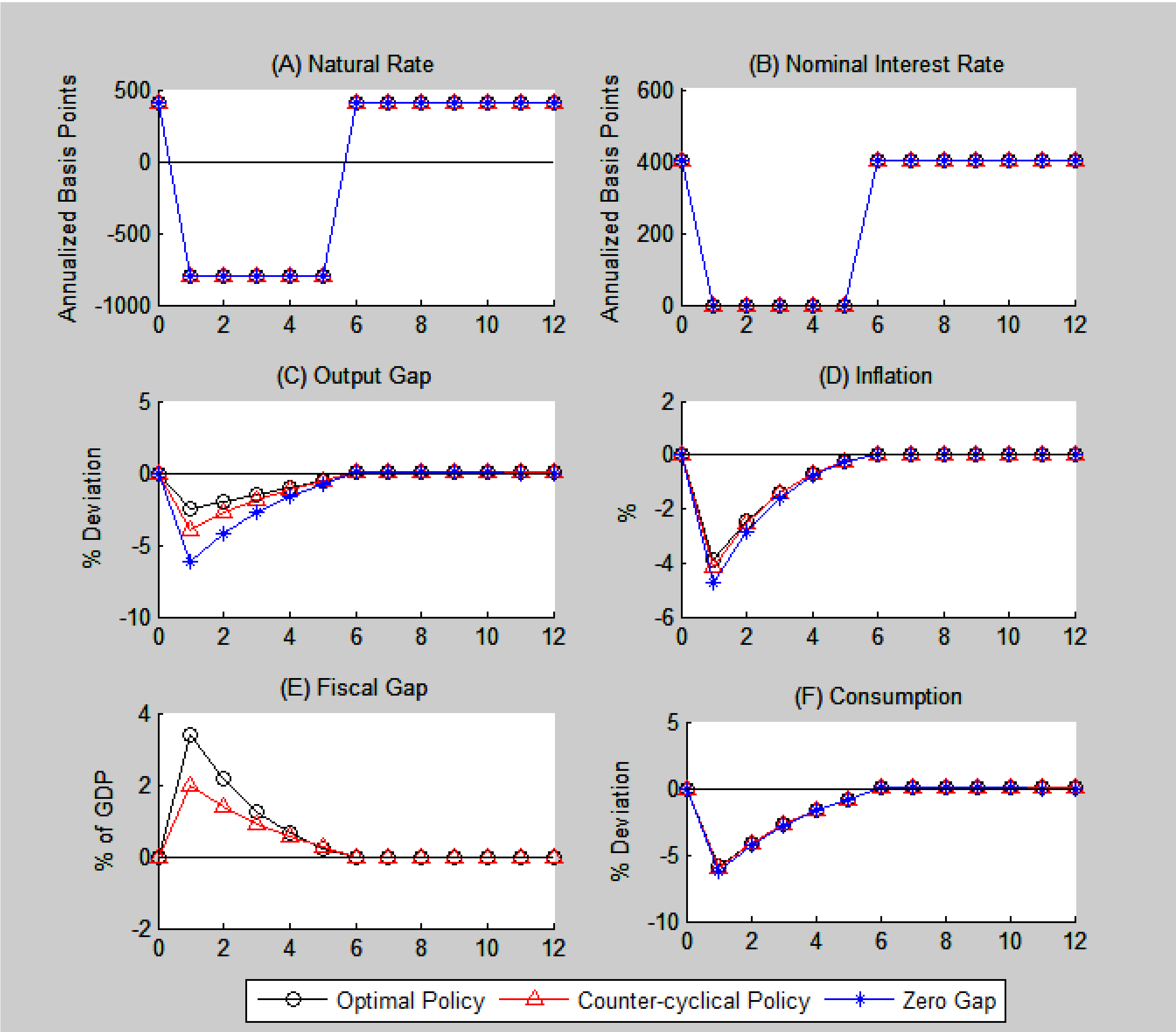


Figure 2 shows the response of regional differentials to a shock to demand in the South economy that lasts for 5 periods. The shock is constructed to bring the aggregate natural interest rate to -8 percent on an annualized rate. The figure shows the response when: *Optimal* both regions implement optimal cooperative fiscal policy; *Countercyclical* each region sets the fiscal policy gap as a constant negative ratio of the output gap; and *Zero Gap* when the fiscal gap in each region is set to zero. Panel (B) shows the real exchange rate of the South region with positive movement indicating the South economy is experiencing depreciation.

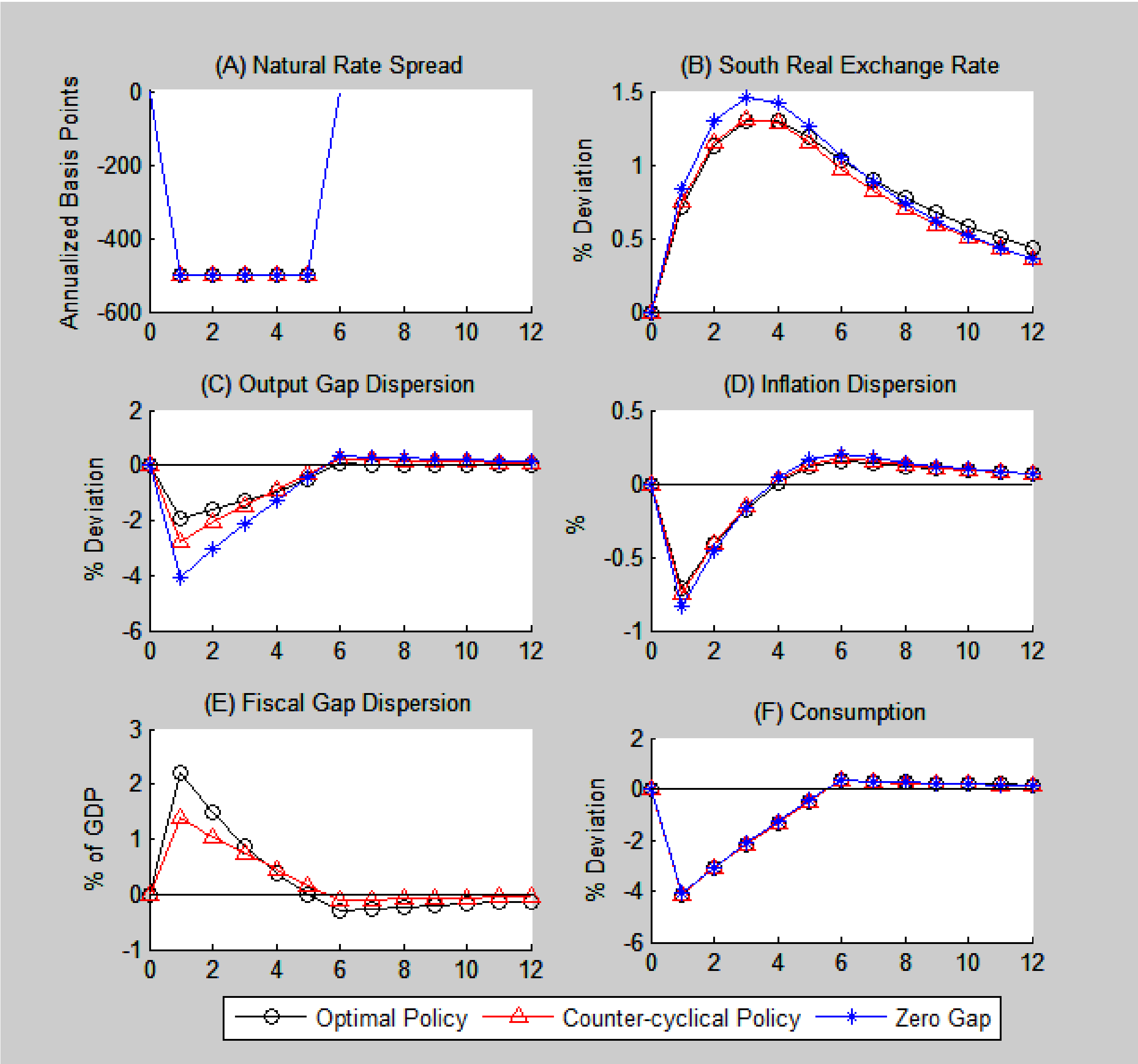


Figure 3 shows the response of each region to a shock to demand in the South economy that lasts for 5 periods pushing the aggregate natural interest rate to -8 percent on an annualized rate. The figure shows the response when: *Optimal,* both regions implement optimal cooperative fiscal policy; *Countercyclical* each region sets the fiscal policy gap as a constant negative ratio of the output gap; and *Zero Gap* when the fiscal gap in each region is set to zero.

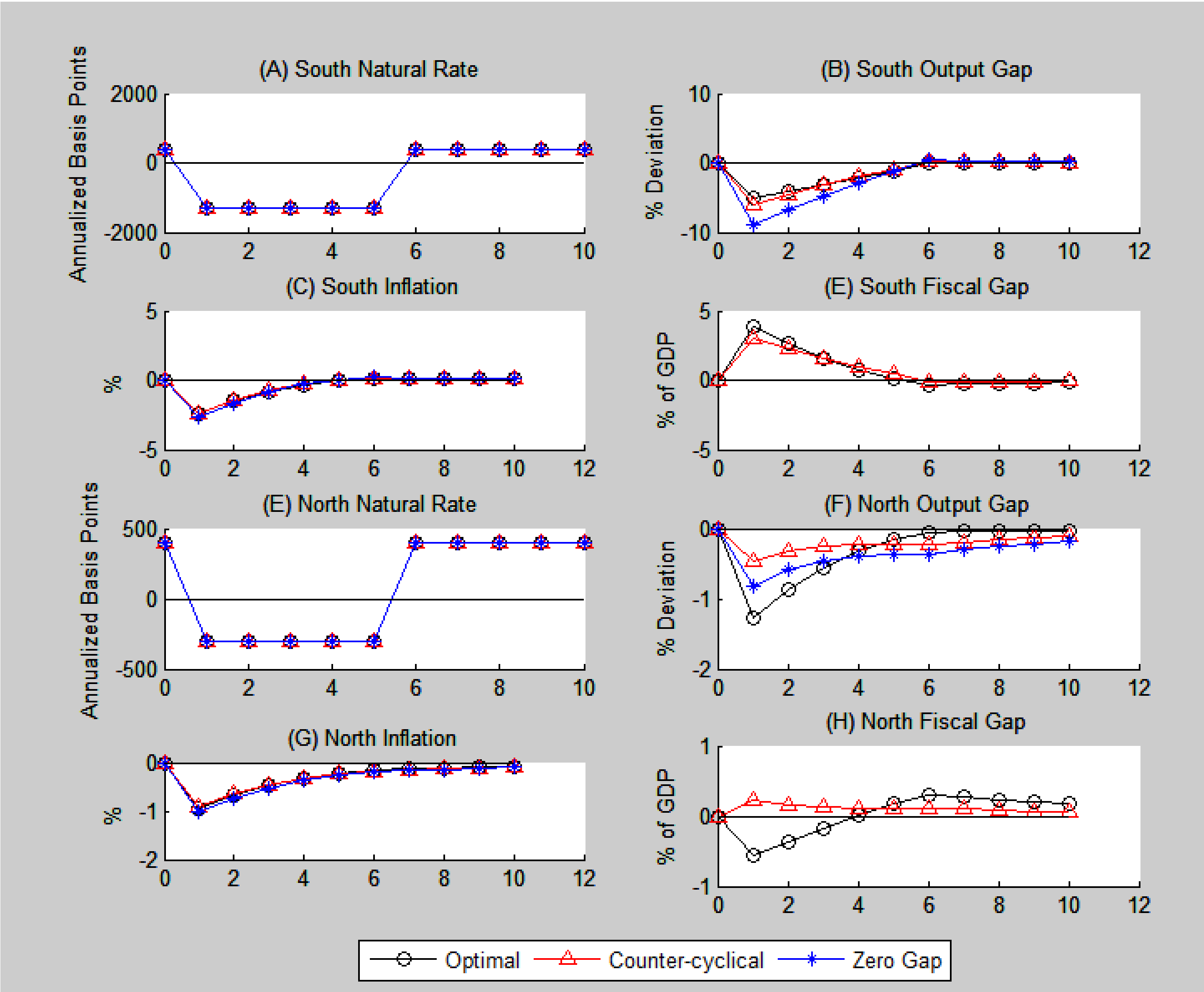


Figure 4 shows the response of the North fiscal policy to a shock to demand in the South economy that pushes the aggregate natural interest rate to -8 percent on an annualized rate. The figure shows the response under optimal cooperative fiscal policy at a variety of parameter values. In all cases except for those displayed in Panel F, the shock lasts for 5 periods. Panel F shows the response of optimal North fiscal policy when the shock lasts for a variety of players.

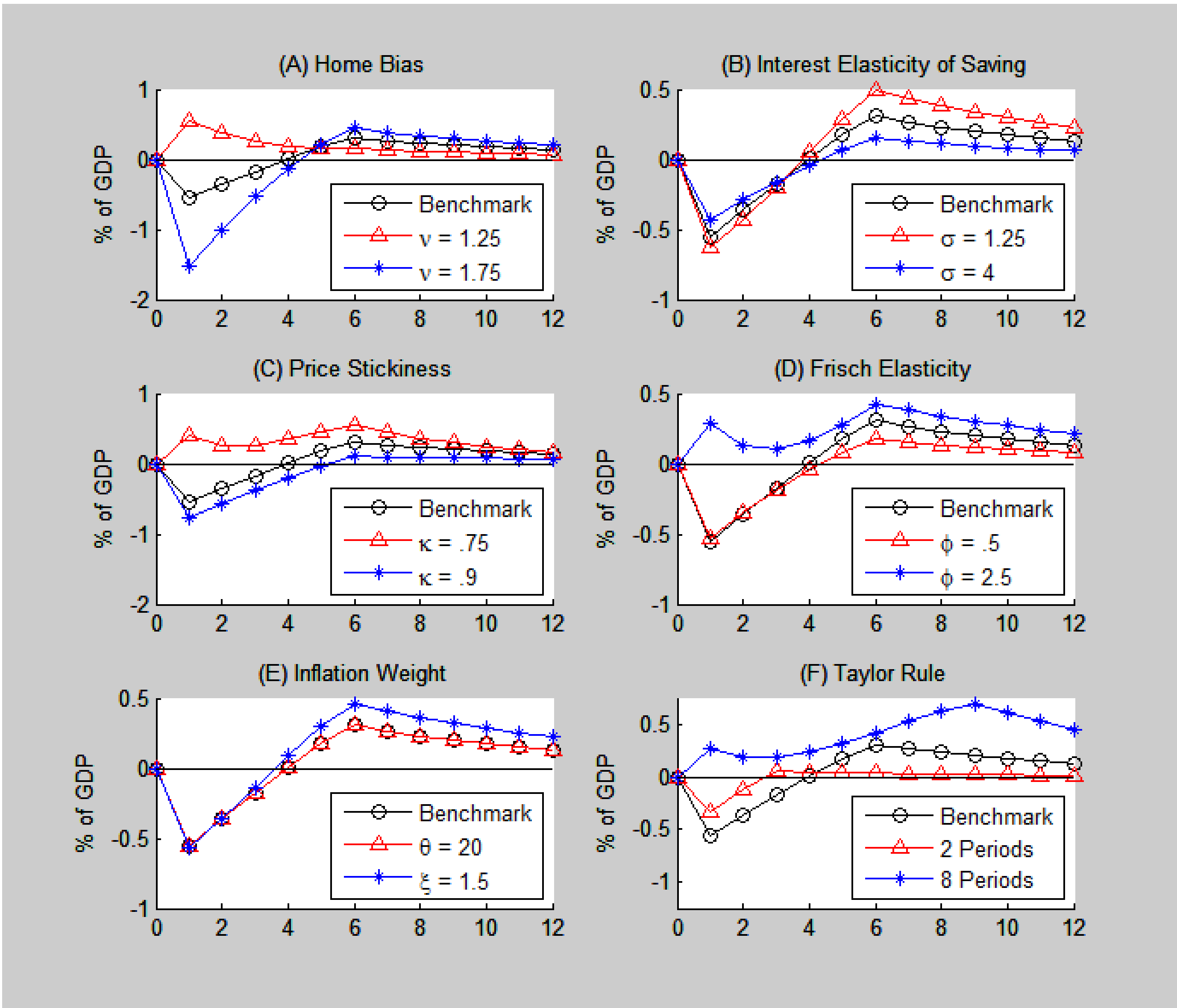


Figure 5 shows the response of North fiscal policy to a shock to demand in the South economy that pushes the aggregate natural interest rate to -8 percent on an annualized rate. In each case, the South fiscal gap is zero. The figure shows the response under optimal constrained cooperative fiscal policy at a variety of parameter values. In all cases except for those displayed in Panel F, the shock lasts for 5 periods. Panel F shows the response of optimal North fiscal policy when the shock lasts for a variety of players.

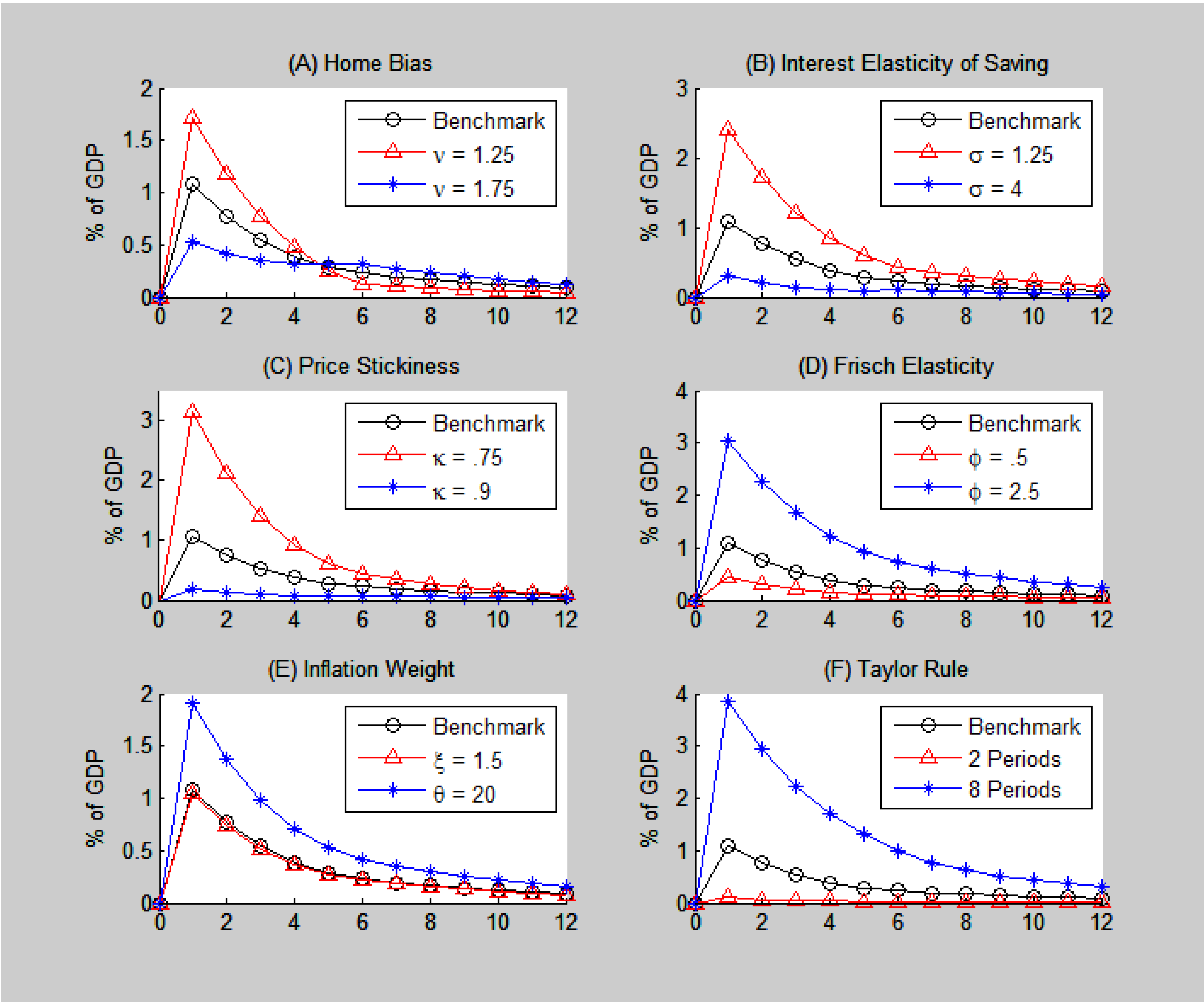
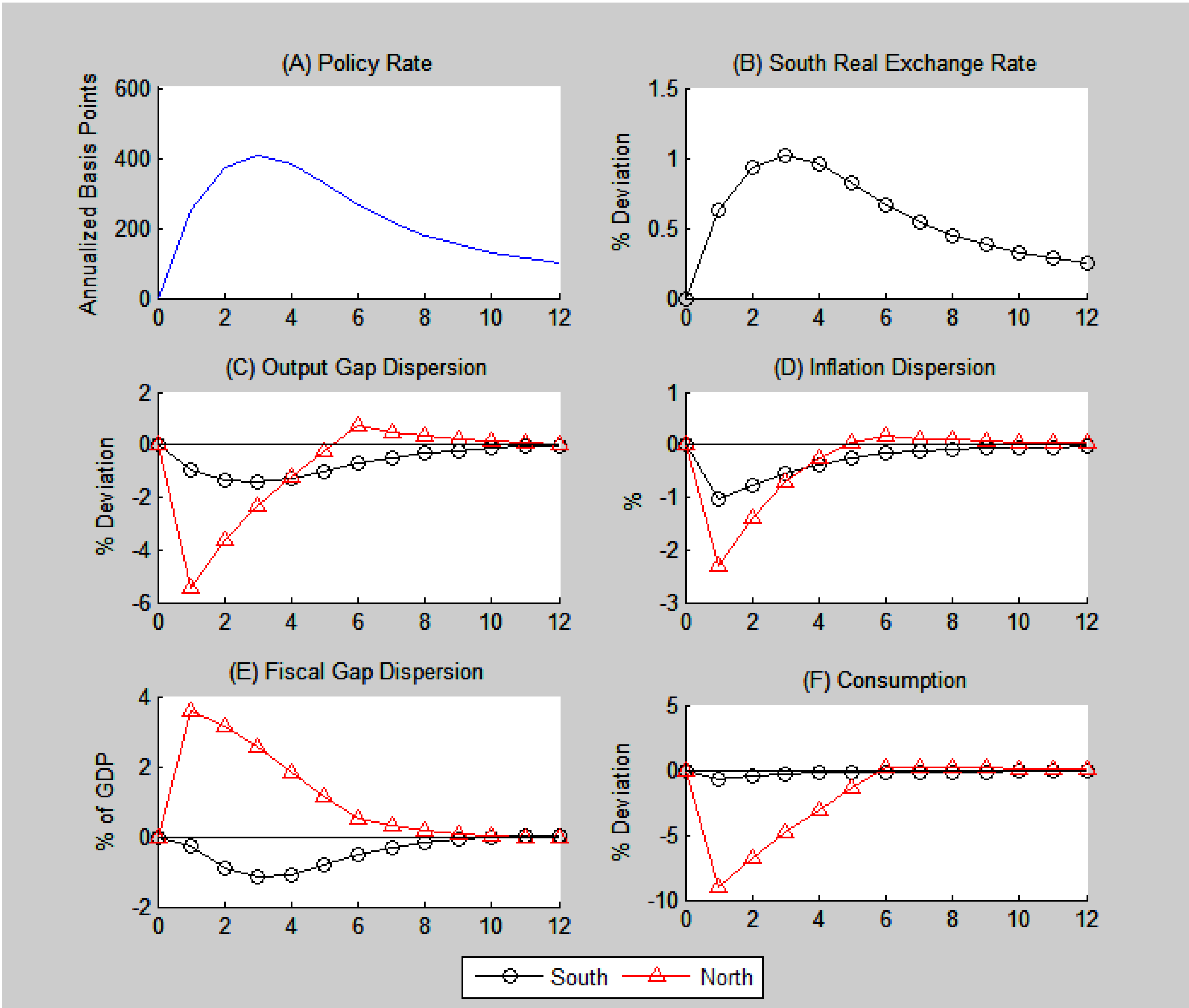


Figure 6 shows the response of North fiscal policy to a shock to demand in the South economy that pushes the aggregate natural interest rate to -8 percent on an annualized rate. In each case, the South fiscal gap is zero. The figure shows the response under optimal constrained cooperative fiscal policy at a variety of parameter values. In all cases except for those displayed in Panel F, the shock lasts for 5 periods. Panel F shows the response of optimal North fiscal policy when the shock lasts for a variety of players.


(A) Policy Rate
Annualized Basis Points
(B) South Real Exchange Rate
% Deviation
(C) Output Gap Dispersion
% Deviation
(D) Inflation Dispersion
%
(E) Fiscal Gap Dispersion
% of GDP
(F) Consumption
% Deviation
South
North